Due Diligence*

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Abstract

Due diligence is common practice prior to the execution of corporate or real estate transactions. We propose a model of the due diligence process and analyze its effect on prices, payoffs, the likelihood of deal completion, and the distribution of completion times. In our model, if the seller accepts an offer, the acquirer has the right to gather information and chooses when to execute the transaction. Our main result is that the acquirer engages in "too much" due diligence relative to the social optimum. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting with no due diligence. The optimal contract involves both a price contingent on execution and a non-contingent transfer, resembling features such as earnest money or break-up fees that are commonly observed in practice.

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1 Introduction

Due diligence is pervasive. It is perhaps most prominent within the context of corporate acquisitions and real estate transactions. However, a period of "discovery" prior to the transfer of ownership and during which the acquirer can gather information extends well beyond these two realms.[1] Practitioners argue that due diligence is critical to ensure a successful transaction.[2] Third-parties (e.g., a financier) often require some due diligence prior to deal completion. But is it economically important? What are the welfare implications? More specifically, how does the acquirer's ability to conduct due diligence prior to executing a deal affect the initial terms, the likelihood of deal completion, the total surplus and how it is divided? In this paper, we propose and analyze a model of the due diligence process to answer these questions.

The due diligence process is an underexplored area in the literature relative to its prominence in practice. This can likely be attributed to a lack of structured data. Empirical work on Mergers and Acquisitions (M&A) often relies on hand-collected data from SEC filings (Boone and Mulherin, 2007; Liu and Officer, 2019).[3] Progress in textual analysis and the availability of new datasets covering private transactions (e.g., PitchBook, Dealogic, and Preqin) suggest the possibility for more empirical work in this area. One of our aims is to provide a theoretical framework for such empirical work.

Due diligence is inherently a dynamic problem. Both the amount and the type of information collected during the process will depend on what the acquirer has learned to date. If a potentially troubling matter is uncovered in the early stages of due diligence (e.g., a pending lawsuit), then the acquirer will spend additional resources investigating the matter

[1] To give one additional example, virtually all retail transactions endow the purchaser with the right to return the item purchased for some period of time. Naturally, the purchaser is likely to learn information about her value for the good during the return period, which influences her decision of whether to exercise the option to return the good.

[2] See e.g., Snow (2011), Lajoux (2010), and Forbes article dated March 27, 2019: "A comprehensive guide to due diligence issues in mergers and acquisitions" (date accessed: August 24, 2020).

[3] More specifically, the "Background to the Offer" section of DEFM14A and Sch 14D1 proxy filings contains rich details about the deal completion process, but it is text-heavy and is not organized in a predefined manner.

or may cancel the deal entirely. If no such issues arise, the transaction will be executed sooner. For practitioners, Snow (2011) describes "the goal (of due diligence) is to make the buyer comfortable enough to go through with the deal and close."

We therefore model due diligence as a stopping problem during which the acquirer uncovers information about the asset being acquired. In our baseline model, there is symmetric imperfect information about bidders' (common) value for the seller's asset and thus whether there are gains from trade. Prior to due diligence, bidders make competing price offers for the asset. If the seller accepts one of the offers then the winning bidder (the *acquirer*) has the right to conduct due diligence and decide when (if ever) to execute the transaction at the bid price. Conducting due diligence takes time, and delay costs are captured by a common discount rate. Thus, the acquirer faces a real option problem during due diligence where the strike price is determined by the winning bid.

The solution to the acquirer's problem is to execute the transaction when the expected value of the asset is above a threshold. This *execution threshold* is increasing in the price: the higher is the price, the more due diligence she conducts prior to execution. A higher execution threshold implies a longer due diligence period and a higher probability of deal failure. As a result, the seller's payoff is not monotonically increasing in the price. Rather, it is initial increasing but eventually decreasing, thereby illustrating the trade-off between a higher payoff conditional on execution versus a longer and less promising due diligence process.

In the unique equilibrium, bidders offer the seller's preferred (interior) price and make a positive expected profit despite being perfectly competitive. Due diligence occurs if and only if the prior belief about the value of the asset is below the execution threshold given the seller optimal price. Above the threshold, the transaction is executed immediately at the highest price such that the acquirer is willing to forgo due diligence. This price is below the acquirer's expected value for the asset, meaning the option to conduct due diligence confers surplus to the acquirer even when it is not exercised.

We perform comparative statics on the speed with which the acquirer learns while conducting due diligence (i.e., the signal-to-noise ratio of the information process). Both the equilibrium price and the execution threshold increase with learning speed, while the likelihood of deal completion falls. Conditional on deal completion, the expected time to completion decreases with learning speed when the prior is low, but increases with learning speed for higher priors. We highlight the implications of these results for empirical work.

We then compare the equilibrium outcome to two benchmarks: the social optimum and no due diligence. Because the equilibrium price is above the seller's reservation value, the acquirer's execution threshold is above the socially optimal one. As a consequence, there is "too much" due diligence and too many deals fail. Relative to the model without due diligence, due diligence increases surplus when the prior is low because without it, the transaction would necessarily fail. But due diligence decreases social surplus for intermediate priors (i.e., near the socially efficient threshold) because the acquirer is too diligent.

In the baseline model, we restrict the space of contracts to prices contingent on execution and assume there is symmetric information. In Sections 3 and 4, we relax these assumptions. With symmetric information, the seller optimal and socially optimal outcomes can simultaneously be achieved by enriching the contract space to allow for upfront, non-contingent transfers. Setting the contingent price equal to the seller's reservation value causes the acquirer to fully internalize the cost of delaying execution, while the upfront transfer allows the seller to extract all the surplus. In contrast, enriching the contract space to include deadlines does not fully resolve the distortion.

When the seller is privately informed of the asset value, agreeing to a contract can signal information to the acquirer, which in turn can influence the acquirer's strategy during due diligence. We first demonstrate that any separating equilibrium involves no trade. The intuition is that if the type is perfectly revealed by the contract, then there is nothing for the acquirer to learn from due diligence. Hence, the acquirer should execute immediately or never. However, if the acquirer executes immediately and a high type is willing to accept the contract, then a low type will strictly prefer to accept it.

Using standard equilibrium refinements, we argue that the high-type optimal equilibrium is focal. This equilibrium involves full pooling and has properties similar to the baseline model. In particular, the contingent price is above the socially optimal level thereby inducing the acquirer to conduct more than the socially optimal level of due diligence. This result is despite the fact that the seller captures all of the surplus from the transaction via the upfront transfer. The intuition is that an inefficiently high contingent price allows a high-type seller to profitably extract some of the information rents from a low-type seller, despite sacrificing total surplus.

We consider several other variations of the baseline model. First, we allow due diligence to begin prior to an offer being accepted. We provide a sufficient condition under which the timing of due diligence relative to offer acceptance is irrelevant. This result emphasizes that the crucial assumption is that the acquirer has the option to conduct due diligence after an offer is accepted, not that this option is necessarily exercised. We then analyze the model with common knowledge of gains from trade. In this case, the aforementioned sufficient condition is violated and the seller prefers that due diligence be performed before accepting an offer. We map these two cases to different types of acquisitions and discuss the empirical implications.

Next, we consider the case in which due diligence requires the acquirer to incur a flow cost. A two-threshold equilibrium emerges in which the acquirer terminates the transaction at a lower threshold after uncovering sufficiently negative information about the asset. Similar to the baseline model, because the acquirer does not internalize the seller's payoff, both thresholds are inefficiently high. Allowing for a break-up fee can improve the outcome. In the absence of discounting, break-up fees are isomorphic to upfront transfers and either can be used to simultaneously implement the revenue-maximizing and socially optimal outcome. Finally, we consider a setting in which the acquirer finances the transaction with (risky) debt from a risk-neutral financier. We show that the option to default reduces the acquirer's incentive to conduct due diligence. As a result, the seller prefers the acquirer use debt to finance the transaction and, if necessary, should be willing to provide "seller financing," which is a common practice in the sale of private firms (Jansen, 2020).

The remainder of the paper is structured as follows. Section 1.1 discusses the due diligence process in practice, and Section 1.2 relates our work to the literature. Section 2 presents the baseline model and its results. Section 3 shows how upfront transfers can help achieve the socially efficient outcome while deadlines cannot. Section 4 studies the model with asymmetric information. Section 5 solves several extensions of the baseline model and discusses their implications. Section 6 concludes. Proofs are found in the Appendix.

1.1 Due Diligence In Practice

A key assumption of our model is that the acquirer has the right to conduct due diligence after agreeing to terms with the seller, which comports with what we observe in practice. There are good economic reasons for this timing when considering the information acquisition and provision costs, though we will largely abstract from such costs in our analysis.

According to Lajoux (2010), "Some buyers spend millions of dollars identifying every possible risk before signing on the dotted line." For mergers in the financial sector, Cole et al. (2016) document that expenditures on accounting and legal firms alone average 47 basis points of the transaction value. This figure does not include internal costs or fees paid to consulting firms or investment banks. Due diligence is also significant in terms of the time to completion: it takes an average of 134 days to complete a merger after publicly announcing it (Offenberg and Pirinsky, 2015). Without the terms of a deal in place, potential acquirers will be unwilling to invest resources needed to complete the due diligence process in a timely manner.

Providing access to proprietary information may also impose strategic costs on the seller. A strategic bidder may use information gathered during due diligence to become more competitive with the target in the case the deal fails (see e.g., Marquardt and Zur, 2015; Wangerin, 2019). A recent example of this behavior occurred in 2019 when Urban Outfitters walked away from a deal to acquire the apparel subscription service Le Tote, after months of due diligence during which Urban Outfitter's executives visited Le Tote's warehouses multiple times to gain an understanding of its business model. Subsequently, Urban Outfitters launched its own apparel subscription service, prompting Le Tote to file a lawsuit against Urban Outfitters alleging a breach of the non-disclosure agreement.[4]

The right to conduct due diligence after terms have been negotiated does not preclude due diligence prior to negotiations. Indeed in M&A, a non-trivial portion of the information gathering takes place during a pre-public phase.[5] Lajoux (2010), Marquardt and Zur (2015), and Wangerin (2019) describe the due diligence that is performed during those deals. Figure 1 illustrates this process. After demonstrating interest and signing a confidentiality agreement, bidders performs due diligence using data supplied by the firm. This phase of a deal is known as the pre-public phase.

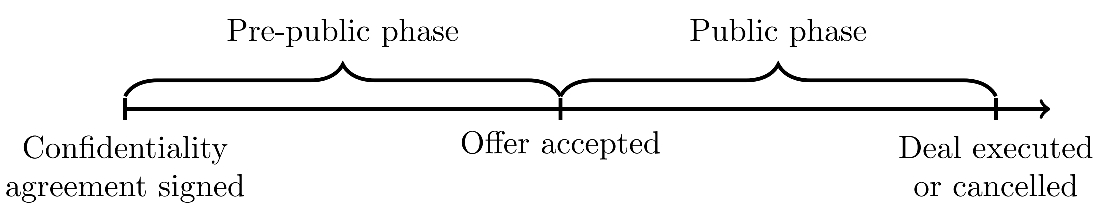


FIGURE 1: Different Phases of an M&A of a Company.

After this pre-public phase, the target firm holds an auction or negotiates bilaterally with interested bidders. During the public phase, the acquirer performs additional due diligence by verifying financial information, legal information, existing obligations to suppliers and customers, the state of physical assets, etc. A significant fraction of deals are canceled during the public phase: 1/8 of deals that were publicly announced between 1986 and 2018 were ultimately canceled (Heath and Mitchell, 2020).

Failure to conduct proper due diligence can have severe financial consequences for the acquirer. A recent example is the Wirecard scandal. In 2019, Wirecard issued \$1bn worth of convertible debt to SoftBank, which it then sold on to investors. A year later Wirecard went bankrupt rendering the debt worthless. An undetected accounting fraud—a non-existing

[4]See Financial Times article dated June 2, 2020: "Is a M&A NDA really just a shadow non-compete?" (date accessed: August 28, 2020)

[5]See Hansen (2001) and Boone and Mulherin (2007) for discussion of the takeover process in M&A.

cash account—had left a €1.9bn hole the company's balance sheet.[6] Another example is Hewlett-Packard's acquisition of Autonomy in 2011. One year after the acquisition, Hewlett-Packard wrote down \$8.8bn in the value of the \$11bn acquisition after accounting irregularities that predated the acquisition were discovered. Hewlett-Packard said that no red flags were raised during due diligence.[7]

Due diligence is also pervasive in private equity and real estate. In private equity, due diligence involves a significant amount of information production because the target has not been subject to the same public scrutiny or disclosure requirements as publicly traded companies. According to data from PitchBook, over 40% of completed private equity deals took more than 15 weeks to close. In residential real-estate transactions, the purchaser typically retains the option to terminate the contract pending the review of seller disclosures, surveys, and inspections, all of which are forms of due diligence. Commercial real estate deals almost always involve an extended due diligence phase that can last for months. The typical due diligence checklist for commercial real estate transactions includes title and zoning verification, tenant and lease matters, existing and potential legal claims, insurance claims, and physical property inspection (Brueggeman and Fisher, 2019, chap. 13).

1.2 Related Literature

Our work relates to the literature on auctions of real options pioneered by Board (2007) and Cong (2018, 2020). We will argue that due diligence expands the scope of this literature. That is, if the acquirer has the right to conduct due diligence prior to execution, then the asset "acquired" when an offer is accepted is in fact a real option. However, because due diligence is (at least) partly to overcome adverse selection (see the discussion in the previous subsection), the information and payoff structure of our model also fundamentally differs from the existing literature. In particular, we analyze a common-value signaling model with

[6]See Financial Times article dated June 22, 2020: "SoftBank executives set to lose profits from Wirecard trade" (date accessed: August 24, 2020).

[7]See Financial Times article dated November 20, 2012: "HP takes \$8.8bn hit over Autonomy" (date accessed: August 24, 2020)

a privately informed seller, whereas this literature considers settings with a private value component in which bidders are privately informed.

DeMarzo et al. (2005) and Gorbenko and Malenko (2011) analyze auctions where the payments to the seller are contingent on the winning bidder's private information, whereas in our model the transfer depends on the acquirer's execution decision. There is also a large literature that studies information acquisition in auctions prior to bidding (Matthews, 1984; Stegeman, 1996; Persico, 2000; Shi, 2012), whereas in our model information is acquired after bidding.

We also contribute to the theoretical literature on M&A.[8] Within this space, our paper is most closely related to the M&A literature that studies the takeover (auction) mechanism, see Bulow and Klemperer (1996), Hansen (2001), Ye (2007), Quint and Hendricks (2018), and Gorbenko and Malenko (2018, 2019). Our contribution is in studying the impact of the due diligence during the public phase, which is especially relevant in takeover auctions (Marquardt and Zur, 2015; Wangerin, 2019).

Our work also relates to the literature on product returns and refund policies (Davis et al., 1995; Courty and Hao, 2000; Matthews and Persico, 2007; Inderst and Ottaviani, 2013; Krähmer and Strausz, 2015). Similar to our paper, this literature explores settings in which the buyer can learn and decide whether to trade after the terms of the deal have been agreed upon. An important distinction of our paper is that the information acquisition process is dynamic, which we have argued is fundamental to due diligence. This distinction also leads to novel insights that cannot be captured in a static setting.

There is a key timing difference in this paper compared to earlier work by Daley and Green (2012, 2020) where learning takes place before the buyer and seller have agreed to terms and ends as soon as the seller accepts an offer. In this paper, the buyer has the option to continue to learn by performing due diligence after terms of the deal have been set. The difference in timing leads to fundamentally different predictions. Perhaps most notable among them is that inefficient delays can occur even with symmetric information

[8] See Betton et al. (2008) and Eckbo et al. (2019) for excellent reviews of this literature.

and common knowledge of gains from trade (see Section 5.2). Whereas if learning ends once an offer is accepted, trade is both fully efficient and immediate with symmetric information and common knowledge of gains from trade.

2 Baseline Model

There is one seller and multiple competitive bidders. The seller owns an asset, which is either of high or low value (also referred to as type), denoted by $\theta \in \Theta = \{L, H\}$. Asset type is unknown to all players. Bidders have a common value for the asset, V_θ, and the seller's reservation value for the asset is k. We assume there are gains from trade if and only if the asset is high value, $V_H > k \geq V_L$, and normalize $V_L = 0$. All agents are risk-neutral and discount cash flows at rate r.

The model takes place in continuous time. At time $t = 0$, the seller holds an auction for the asset. Each bidder makes a price offer and the seller selects the winning bid. After the auction, the winning bidder (henceforth, the *acquirer*) can perform due diligence and decides when (if ever) to execute the transaction and complete the deal. During due diligence, the acquirer gathers information about the type of the asset. If the winning bid is P and the acquirer executes the transaction at date τ, then the seller's (net) payoff is $e^{-r\tau}(P - k)$ and the acquirer's payoff is $e^{-r\tau}(V_\theta - P)$. If the transaction is never executed then all players' payoffs are zero.

2.1 Learning

The seller and bidders have a common prior $q_0 \in (0, 1)$, which is the probability they assign to $\theta = H$. During due diligence, the acquirer observes information about the asset's type from a Brownian information process

$$dX_t = 1_{\{\theta=H\}}dt + \frac{1}{\phi}dB_t, \tag{1}$$

where B_t is a standard Brownian motion on the canonical probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and ϕ is the signal-to-noise ratio or the "speed" at which the acquirer learns during due diligence. A higher ϕ makes the information process less noisy and therefore more informative about θ per unit of time. The information process $\{X_t\}_{t\geq 0}$ generates a filtration $(\mathcal{H}_t)_{t\geq 0}$. Let q_t denote the probability the acquirer assigns to $\theta = H$ conditional on information acquired up to date t. For computing q_t, X_t is a sufficient statistic for the entire path.[9] Therefore, q_t can be computed from Bayes' rule as

$$q_t = \frac{q_0 f_t^H(X_t)}{q_0 f_t^H(X_t) + (1-q_0) f_t^L(X_t)},$$

where f_t^θ denotes the pdf of X_t conditional on θ. Using Ito's lemma, the evolution of q_t is given by

$$dq_t = \phi^2 q_t(1-q_t)\left(dX_t - q_t dt\right).$$

Notice that $\phi\left(dX_t - q_t dt\right)$ is the increment of a standard Brownian motion on the probability space $(\Omega \times \Theta, \mathcal{F} \times 2^\Theta, \mathcal{P} \times \upsilon)$, where υ is the measure over Θ defined implicitly by q_0.

2.2 Strategies and Equilibrium Concept

The game can be divided into two stages. In the first stage, bidders simultaneously make offers to the seller and the seller decides which offer to accept. In the second stage, the acquirer conducts due diligence until she deems it optimal to execute the transaction. The second stage is a proper subgame, which we refer to as the *due diligence subgame*.

Suppose the winning bid price is P. In the due diligence subgame with initial belief q,

[9] This statement follows from Girsanov's theorem upon observing that the Radon-Nikodym derivative for a change in the measure (over paths) conditional on $\theta = H$ to the measure conditional on $\theta = L$ depends only on X_t.

the acquirer chooses a stopping time to maximize her expected discounted payoff

$$F_B(q|P) = \sup_{\tau} \mathbb{E}_q \left[e^{-r\tau}(V(q_\tau) - P) \right],$$

where $V(q) = qV_H + (1-q)V_L$. The acquirer's strategy in the due diligence subgame is therefore a collection of stopping times, which are indexed by the winning bid price with elements denoted by $\tau(P)$.

The seller must take into account the acquirer's strategy in the due diligence subgame when deciding which offer to accept. Let $F_S(q|P)$ denote the seller's expected payoff from accepting an offer of P, starting from belief q:

$$F_S(q|P) = \mathbb{E}_q \left[e^{-r\tau(P)}(P - k) \right].$$

Clearly, the seller will reject any offer below k and the bidder never completes the transaction if the price is higher than V_H. Therefore, we can restrict attention to bids in the interval $[k, V_H]$. Our equilibrium concept is subgame perfect Nash equilibrium, henceforth referred to simply as *equilibrium*.[10]

2.3 Equilibrium Analysis

We solve for the equilibrium by backward induction. Given any price P, the acquirer faces a stopping problem of when to complete the transaction. The solution to this problem is to complete the transaction as soon as the belief exceeds a threshold.

Lemma 1 (Acquirer-Optimal Execution)**.** *Given any price $P \in [k, V_H]$, the acquirer completes the deals as soon as q_t exceeds*

$$b(P) = \frac{1}{1 + \frac{1-\underline{q}(P)}{\underline{q}(P)} \times \frac{u-1}{u}} > \underline{q}(P) \tag{2}$$

[10]The role of subgame perfection is to require the acquirer to play optimally in the due diligence subgame for any possible winning bid and not just the bid that is accepted on the equilibrium path.

where $u = \frac{1}{2}(1 + \sqrt{1 + 8r/\phi^2})$ *and* $\underline{q}(P) = \frac{P - V_L}{V_H - V_L}$*. That is,* $\tau^*(P) = \inf\{t > 0 | q_t \geq b(P)\}$*.*

The optimal acquisition threshold depends on the product of two terms. The first term depends only on the belief such that the expected value of the asset is equal to the price, $\underline{q}(P)$, which is increasing in P. This belief is akin to the strike price on a call option: for beliefs above $\underline{q}(P)$ the option is in the money. The second term depends only on $\gamma \equiv \phi^2/r$, which is the speed of learning per unit cost of time. The higher is γ, the greater is the option value from due diligence and the higher is the execution threshold. As $\gamma \to 0$, the value of conducting due diligence goes to zero and $b(P) \to \underline{q}(P)$. Figure 2 illustrates the acquirer's value function, the optimal execution threshold, and its relation to $\underline{q}(P)$.

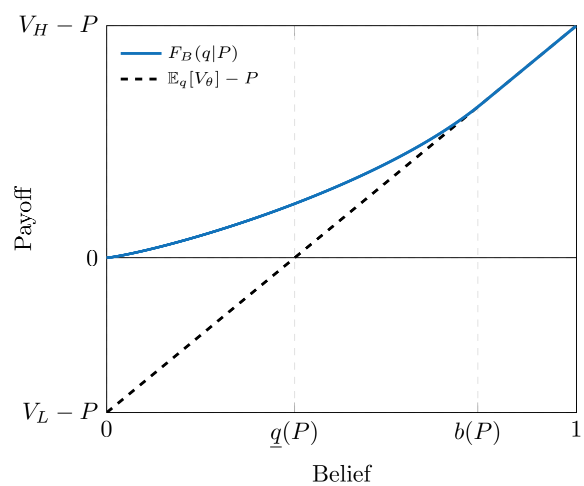


FIGURE 2: Solution to the Acquirer's Problem in the Due Diligence Subgame.

Having characterized the acquirer's strategy in the due diligence subgame, consider the first stage wherein the price is determined. Because bidders are identical and perfectly competitive, the equilibrium price maximizes the seller's expected payoff. Clearly, a higher price is good for the seller conditional on execution. However, because a higher price implies a higher execution threshold, it also implies more delay and a higher chance of deal failure. When the price is low (i.e., close to the seller's reservation value), the marginal cost of a higher price is relatively small and outweighed by the higher payoff conditional on execution.

For higher prices, the marginal cost of a higher price outweighs the marginal benefit. As a result, the seller's payoff is initially increasing in P and eventually decreasing in P.

To characterize the seller-optimal price, let $P_0(q)$ denote the highest price at which the acquirer is willing to forego due diligence and execute the transaction immediately given the belief is q. In order to induce immediate execution, the seller must provide some rents to the acquirer. Thus, $P_0(q) < V(q)$. A closed-form expression for P_0 can be obtained by inverting $b(P)$ from equation (2) to get

$$P_0(q) = \frac{(1-q)uV_L + q(u-1)V_H}{u-q}.$$

Next, consider the hypothetical stopping problem in which the seller chooses when to accept $P_0(q_t)$:

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau}(P_0(q_\tau) - k) \right]. \tag{HSP}$$

The value under the solution to this problem provides an upper bound on the seller's equilibrium payoffs. The solution necessarily involves a belief threshold above which it is optimal to stop. The following assumption says that it is never optimal to stop below the threshold.

Assumption 1. *There exists a* $q^* \in (0,1)$ *such that the solution to* (HSP) *is* $\tau_{HSP} \equiv \inf\{t \geq 0 : q_t \geq q^*\}$.

Assumption 1 is not directly about primitives of the model. The next lemma provides sufficient conditions on primitives for it to hold.

Lemma 2. *Fixing all other parameters, Assumption 1 is satisfied if either:*

(i) $k \geq \bar{k}$ *for some* $\bar{k} \in (V_L, V_H)$.

(ii) $\gamma > \bar{\gamma}$ *for some* $\bar{\gamma} > 0$.

Assumption 1 can fail if k is small enough and γ is neither too large, nor too small. Our numerical analysis suggests the region of the parameter space in which Assumption 1 can fail is rather small: $\bar{k}$ is less than 5 percent of V_H across all γ.[11] However, it necessarily fails if there is common knowledge of gains from trade (i.e., $k < V_L$). We analyze this case in Section 5.2, but assume that Assumption 1 holds until then.

Lemma 3 (Seller-Optimal Price)**.** *Let $P^* \equiv P_0(q^*)$. The seller optimal price, denoted $P_S(q)$, is given by*

$$P_S(q) = \begin{cases} P^* & \text{if } q \leq b(P^*) \\ P_0(q) & \text{if } q > b(P^*) \end{cases}$$

For $q \geq b(P^*)$, the seller prefers an offer that induces the acquirer to forego due diligence and execute the transaction immediately. For $q < b(P^*)$, the seller prefers the acquirer to conduct due diligence with the hope of selling for P^* in the future rather than settle for $P_0(q)$ today. Naturally, competition among bidders drives the winning offer to the seller's preferred price.

Proposition 1. *There exists a unique equilibrium. In it,*

(i) The winning offer is the seller-optimal price, $P_S(q_0)$.

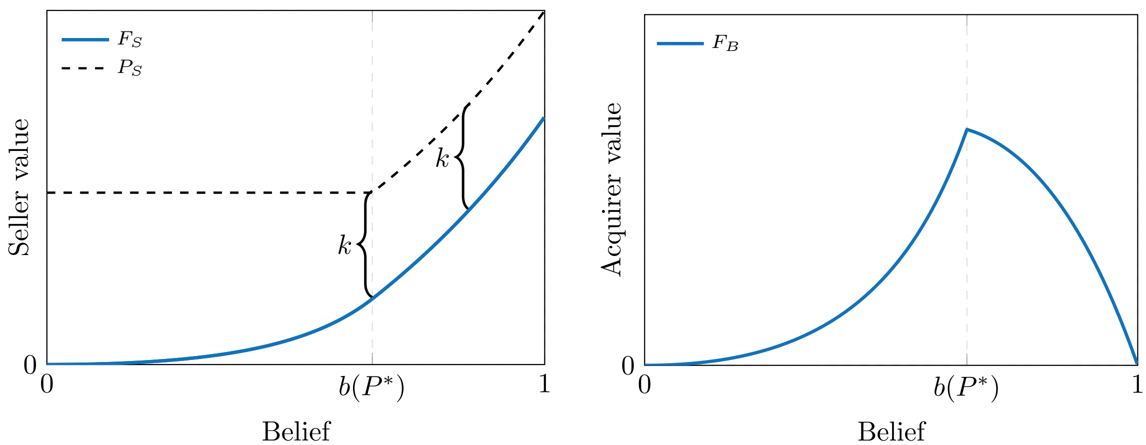
(ii) In the due diligence subgame, the acquirer plays according to $\tau^*(P)$.

(iii) There is a period of due diligence if and only if $q_0 < b(P^*)$.

Figure 3 illustrates the equilibrium payoffs and prices as they depend on the initial belief. Notice that the seller-optimal price is constant for all $q_0 < b(P^*)$, corresponding to the region in which due diligence takes place. As a result, the seller's equilibrium payoff is the same as the payoff in (HSP). This payoff equivalence has two important implications.

First, play is renegotiation proof along the equilibrium path—the seller does not have an incentive to try to renegotiate the price up or down in response to information being

[11] Our proof of Lemma 2 establishes that $\bar{k}$ depends only on γ and V_H.



(a) Equilibrium Price and Seller Value

(b) Acquirer Value

FIGURE 3: Equilibrium Prices and Payoffs as a Function of the Belief.

revealed. Second, whether due diligence takes place before or after (or during) bidding is payoff irrelevant. We formally demonstrate this result in Section 5.1, where we extend the model to allow for dynamic bidding. Thus, the key assumption for our results is that the acquirer has the option to conduct (more) due diligence after her offer is accepted. If bidders can conduct due diligence prior to or during negotiations, then the option to conduct further due diligence after an offer is accepted may not be exercised. However, the equilibrium payoffs remain unchanged.

2.4 Implications

How does the acquirer's ability to conduct due diligence affect equilibrium quantities? Does due diligence improve efficiency or lead to unnecessary delays? To answer these questions, we first utilize the Martingale property of the belief process to derive expressions for quantities of interest. We then consider the effect of an increase in ϕ and compare the equilibrium outcome to two benchmarks.

Lemma 4 (Deal Completion and Time to Completion)**.** *Suppose that $q_0 < b(P^*)$. Let τ^* denote the time at which the deal is completed.*

(i) The probability that the deal is eventually completed is

$$\mathbb{P}(\tau^* < \infty) = \frac{q_0}{b(P^*)},$$

(ii) Conditional on the deal being completed, the expected time to completion is

$$\mathbb{E}\left[\tau^* | \tau^* < \infty\right] = \frac{2}{\phi^2}\left(\ln\left(\frac{b(P^*)}{1-b(P^*)}\right) - \ln\left(\frac{q_0}{1-q_0}\right)\right).$$

Speed of Learning Recall that ϕ denotes the signal-to-noise ratio and measures the "speed" with which the acquirer learns during due diligence. The following proposition summarizes how ϕ affects equilibrium quantities.

Proposition 2 (Speed of Learning)**.** *Suppose that $q_0 < b(P^*)$. An increase in the acquirer's speed of learning, ϕ, leads to*

(i) A higher equilibrium price,

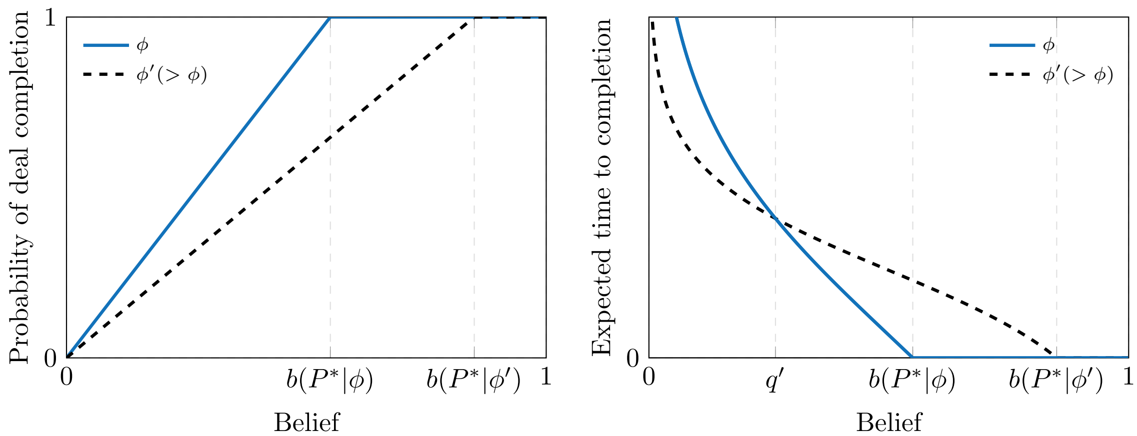
(ii) A lower probability of deal completion, and

(iii) A decrease (increase) in the conditional expected time to completion for $q_0 < q'$ ($q_0 > q'$) for some $q' \in (0, b(P^*))$.

Figure 4 illustrates the impact of learning speed on deal completion and the expected time to completion. Increasing ϕ has two effects on completion time. First, $b(P^*)$ increases, which leads to longer time to completion.[12] Second, learning occurs faster, which shortens time to completion. The first effect dominates near the execution threshold, while the second effect dominates further away from the threshold.

Proposition 2 has empirical implications, which could be tested either by developing proxies for ϕ or by using the deal premium as a proxy for ϕ while controlling for value parameters. To illustrate the latter approach, note that the deal premium (i.e., $P^* - k$)

[12]From Lemma 1, b is increasing in both ϕ and P (see Lemma 1), and P^* is itself increasing ϕ by (i).



(a) Probability of Deal Completion (b) Conditional Expected Time to Completion

FIGURE 4: Impact of Learning Speed on Probability of Deal Completion and Time to Completion.

is increasing in ϕ. Thus, Proposition 2 predicts that, holding the value parameters fixed, deals with a higher premium will be more likely to fail. Moreover, conditional on the deal being completed, the time to completion is non-monotone in the premium. In particular, among deals that are completed relatively quickly (i.e., $q_0 > q'$), a higher premium should be associated with a longer time to completion. However, among deals that take a relatively long time to complete (i.e., $q_0 < q'$), a higher premium should be associated with a shorter time to completion.

No Due Diligence Consider a benchmark without due diligence: if the seller accepts the bidder's offer then the transaction is immediately executed. In this case, the seller's expected payoff is increasing in the price. As a result, competitive bidders offer a price equal to the expected asset value and the seller accepts the offer when it is above her reservation value.[13] Without the ability to conduct due diligence, the acquirer makes zero profit.

Corollary 1. *The acquirer's ability to perform due diligence allows her to extract a positive surplus in equilibrium:* $F_B(q_0|P_S(q_0)) > 0$ *for all* $q_0 \in (0,1)$.

Note that this result holds regardless of whether the acquirer actually conducts any due

[13] This is also the outcome that obtains in the limit as $\gamma \equiv \phi^2/r \to 0$.

diligence along the equilibrium path. The mere option to conduct due diligence is what facilitates surplus extraction.

Figure 5 plots the social surplus and seller value both with and without due diligence. The figure suggests that due diligence improves surplus for low beliefs but hinders it for intermediate beliefs. The following proposition formalizes this result.

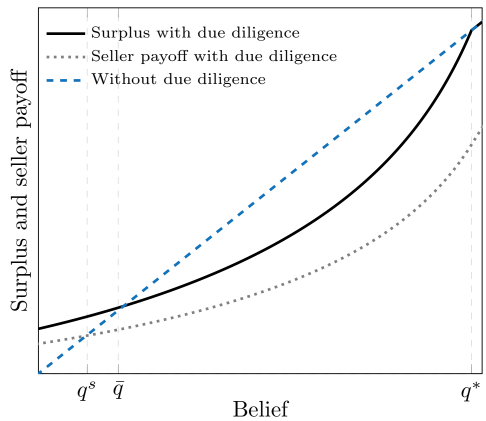


FIGURE 5: Impact of Due Diligence on Surplus. Note that without due diligence, the total surplus and the seller's payoff coincide.

Proposition 3 (Social Surplus). *There exists* $\bar{q} < b(P^*)$ *such that*

(i) For $q_0 < \bar{q}$*, due diligence increases social surplus.*

(ii) For $q_0 \in (\bar{q}, b(P^*))$*, due diligence decreases social surplus.*

(iii) For $q_0 \geq b(P^*)$*, due diligence does not change social surplus.*

Intuitively, allowing for due diligence increases surplus when the deal would necessarily fail without it. However, because the acquirer is too diligent relative to the social optimum (Proposition 4), allowing for due diligence when it is (close to) socially optimal to execute decreases surplus.

Social Optimum Consider a social planner who does not know the asset's type, but has the ability to learn by observing the information generated during the due diligence. The planner's problem is to choose a stopping time to solve

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau}(V(q_\tau) - k) \right].$$

Notice that the planner's problem is identical to the acquirer's problem except that the strike price of the option is the seller's reservation value rather than the winning bid.

Proposition 4 (Social Optimum)**.** *The socially optimal execution threshold is $b(k)$, which is strictly less than $b(P^*)$. Therefore, in equilibrium, the acquirer conducts "too much" due diligence and the probability of deal failure is too high relative to the social optimum.*

Intuitively, this result derives from the fact that the acquirer only captures part of the surplus, and therefore does not internalize the full cost of delay or deal failure. To induce efficient execution, the price paid conditional on executing the transaction should be set equal to k. But if the price is k, then the seller captures none of the surplus. Competition among bidders drives the price above k, which in turn leads to inefficient execution.

3 Seller Optimal Mechanism

In this section we consider two enrichments of the contract space: deadlines and upfront transfers. The main takeaway is that deadlines do not resolve the distortion identified in Proposition 4. Whereas, if we allow for upfront transfers, then the seller optimal mechanism achieves the socially efficient outcome.

3.1 Deadlines

Section 2.4 examined the model without due diligence. There, in contrast to the baseline model, seller surplus and total surplus coincide and the social optimum is achieved for q_0 high enough. An agreed upon deadline represents a less extreme curtailment of the duration of the due diligence process. Could allowing this contract feature similarly enable the seller to extract the socially optimal surplus?

To investigate, we enrich the contract space as follows: a *deadline contract* consists of a price P and a time-horizon $T \in (0, \infty)$. If the deal does not get executed on or before T, then the agreement is terminated and both parties earn zero.[14]

Introducing a deadline makes the acquirer less patient in the due diligence phase. As a consequence, the highest price at which she is willing to execute increases as the deadline approaches and converges to $\mathbb{E}_q[V_\theta]$ as $T \to 0$. From the seller's perspective, the benefit of a deadline is that it reduces the acquirer's ability to extract surplus from the deal. The cost of a deadline is the (additional) risk that the deal fails.

Proposition 5 (Deadlines)**.** *A deadline contract is never simultaneously seller optimal and socially optimal.*

The intuition is straightforward. When due diligence is socially desirable ($q_0 < b(k)$), a deadline may increase the seller's payoff, but has a positive probability of inefficient termination. When due diligence is socially undesirable, a mere deadline is not sufficiently constraining—*any* period of investigation both diverts surplus from the seller to the acquirer and lowers social surplus.

3.2 Upfront Transfers

We enrich the contract space such that a contract now consists of both an upfront transfer and a price contingent on execution. Doing so allows bidders to transfer surplus to the seller without inefficiently distorting the execution decision. A contract, C, consist of a pair $C \equiv (U, P)$, where $U \geq 0$ is an upfront unconditional transfer and P is the price paid contingent on execution. In this case, we obtain the following result.

[14]Proposition 5 continues to hold if, upon termination, the seller gets any fraction (less than one) of the value from finding a new set of bidders (with belief q_T). The fraction can be interpreted as the expected discount factor associated with finding a new set of bidders to bid for the asset.

Proposition 6 (Upfront Transfers)**.** *The seller optimal mechanism with upfront transfers involves* $(U, P) = (F_B(q_0|k), k)$ *if* $q_0 < b(k)$ *and immediate execution if* $q_0 \geq b(k)$*. In either case, the acquirer's execution threshold is socially optimal and the seller captures the entire surplus.*

In effect, the acquirer purchases the option to buy the firm at a price of k and, in doing so, transfers all of the surplus to the seller through the upfront payment. The contingent price is set so that the acquirer correctly internalizes the social cost and benefit of conducting due diligence.

Proposition 6 is related to Board (2007), who studies the sale of a real option in a private value setting. There are two differences. First, Board (2007), assumes that the seller's reservation value evaporates after the option is transferred to the bidder regardless of whether the option is executed, whereas in our setting the seller foregos his reservation value only if/when the option is executed. Therefore, Board (2007) concludes that the socially optimal mechanism does not involve a contingent transfer. Second, in our model bidders have no private information (so do not earn information rents). Because the seller can extract all of the surplus, there is no tension between revenue maximization and surplus maximization, unlike in Board (2007).

Although allowing for upfront transfers resolves the distortion in our baseline model, they are not a panacea. For instance, as we demonstrate in the next section, when the seller is privately informed, larger contingent transfers serve as an effective screening mechanism. Consequently, the equilibrium contract involves contingent transfers that are larger than is socially optimal, again resulting in "too much" due diligence, even with upfront transfers. In another instance, Board (2007) shows that when bidders have private information about their valuations, the revenue-maximizing mechanism relies too heavily on contingent transfers, inducing the acquirer to inefficiently postpone exercising the option.

4 Asymmetric Information

In this section, assume that the seller knows the asset's type $\theta \in \{L, H\}$, while as before, the type is unknown to bidders. In addition, we allow the seller's reservation value to depend on the asset's type k_θ with $V_H > k_H > k_L > V_L = 0$. As in Section 3, we pay particular attention to contracts, $C = (U, P)$, that specify U as the unconditional upfront transfer and P as the price paid contingent on execution.[15] Let $F_\theta(q|P) = \mathbb{E}_q^\theta \left[e^{-r\tau(P)}(P - k_\theta)\right]$ denote the value to a type-θ seller derived from the due diligence subgame with contracted price P. F_θ differs from F_S in that both the expectation about what the acquirer will learn during due diligence and the reservation value depend on θ.

4.1 Constrained Efficiency

We first revisit the problem of a social planner. As before, the first-best outcome is to execute the transaction immediately if $\theta = H$ and never if $\theta = L$. We refer to the *constrained efficient* outcome as the solution to the problem of a planner who has the same access to information as the acquirer (i.e., the planner does not know θ but can learn about it through due diligence). The planner's problem can be written as $\sup_\tau \mathbb{E}_q \left[e^{-r\tau}(V(q_\tau) - k_\theta)\right]$.

As before, it is socially optimal to delay executing the deal until beliefs reach a threshold, denoted b_{SP}, above which it is socially optimal to execute the deal. Moreover, there exists a unique contingent price $P_{SP} \in (k_L, k_H)$ under which b_{SP} is the acquirer-optimal threshold in the due diligence subgame. That is, $b(P_{SP}) = b_{SP}$.[16]

While a contingent price of P_{SP} induces constrained efficient execution, the difference in seller types creates a hurdle to the implementation of the constrained efficient outcome. Because $P_{SP} < k_H$, the high-type seller earns a loss at the time of execution. To satisfy the high-type seller's participation constraint, this loss needs to be overcome with a transfer from the acquirer. However, when the initial belief is pessimistic, even transferring the acquirer's

[15] In Section 4.3, we consider the case in which the terms of the transaction can depend on the information obtained during the due diligence process X.

[16] Specifically, $b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$, and $P_{SP} = \alpha V_H + (1-\alpha)V_L$, where $\alpha = \frac{k_L - V_L}{(V_H - V_L) - (k_H - k_L)}$.

entire expected surplus is insufficient to compensate for the high type's expected loss.

Proposition 7 (Constrained Efficiency)**.** *There exists a contract* (U, P) *that achieves the constrained efficient outcome and satisfies bidder and seller participation constraints if and only if*

$$q_0 \geq \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{V_H - V_L}\right\}. \tag{3}$$

4.2 Equilibrium

We now analyze the game in which the seller posts a contract $C = (U, P)$. Under this formulation, the model's first stage is a signaling game due to the seller's private information.[17] We therefore modify our solution concept from subgame perfect Nash equilibrium to perfect Bayesian equilibrium (PBE).[18] In equilibrium, for any posted contract C, bidders update their belief about θ from q_0 to $\tilde{q}(C)$ that is consistent with the seller's posting strategy. If accepted, the seller's total expected payoff from $C = (U, P)$ is $U + F_\theta(\tilde{q}(C)|P)$.

As is common in signaling games, there exist many PBE, due to the flexibility afforded to off-path beliefs. Consider, for example, the constrained efficient PBE (assuming (3) is satisfied) in which both seller types pool on the contract with $P = P_{SP}$ and attempt to extract all the surplus, $U = U_{SP}(q_0) \equiv F_B(q_0|P_{SP})$. Since both types post the same contract, bidders do not learn anything from the seller's posting (i.e., $\tilde{q}(U_{SP}(q_0), P_{SP}) = q_0$).

How reasonable is the constrained efficient equilibrium? This question is usually addressed by asking, what "should" bidders believe if an off-path signal is observed? In our case, suppose the seller unexpectedly posts C' consisting of a slightly higher price and a slightly lower upfront. Which type is more willing to make this tradeoff? A higher price induces more due diligence. Because the high type expects better news, increased due diligence is more beneficial (or less detrimental) to the high type than to the low type. What

[17] Our results are unchanged if instead bidders make public contract offers to the seller. While the arguments are slightly more involved in this case, the key is that the seller's choice of contract to accept still serves as a signal of his private information.

[18] See Fudenberg and Tirole (1991, pp. 331-333).

is a "reasonable" belief to hold, then, after observing C'? The relatively weak equilibrium refinement known as *divinity* (Banks and Sobel, 1987) requires only that C' does not lead the bidder to think worse of the seller's type: the belief following C' should be no lower than the prior q_0. But if there is no deterioration of belief from posting C', then it may well be attractive to the high type to do so.

In fact, it can be shown that the constrained efficient PBE satisfies divinity if and only if it maximizes the high-type seller's payoff among all PBE. Partially motivated by this fact, we identify the high-type optimal PBE, hereafter the *HTO equilibrium*. Unlike many signaling games, the high type in our model never does best by separating.

Proposition 8. *In any separating PBE, the transaction is never executed.*

In order to separate from the low type, the high type must post a contract so unfavorable to the bidder that she never executes *even though* the acquirer is sure the seller has a high type asset (otherwise the low type would prefer to imitate the high type). Of course, in a separating equilibrium, the low type does not transact either.[19] Henceforth, we refer to such equilibria, in which the transaction is never executed, as *trivial*.

We conclude our discussion of the connection between refinements and HTO equilibria with the following equivalence.

Proposition 9. *A nontrivial PBE is a HTO equilibrium if and only if it satisfies both divinity and the undefeated criterion (Mailath et al., 1993).*[20]

Below we provide sufficient conditions for the HTO equilibrium to be unique. Numerical results indicate that uniqueness is generic, but it remains unproven.

[19] Moreover, the *intuitive criterion* (Cho and Kreps, 1987)—the canonical refinement used to generate separation in signaling games—has no refining power in our game because the signal in our model (i.e., posting a contract) has no direct cost. For example, no matter how large a price the seller demands, the worst that can happen is that the contract is rejected, resulting in a payoff of zero.

[20] The undefeated criterion is arguably the most well known refinement that is not based on Kohlberg and Mertens (1986)'s notion of *stability* (on which both divinity and the intuitive criterion are based). While neither refinement selects a unique PBE in our model, it is appealing that HTO equilibria are the sole survivors of these refinements with different foundations.

4.2.1 Characterizing the HTO Equilibrium

Proposition 8 implies that a HTO equilibrium must involve at least some pooling. In fact, it involves full pooling.

Lemma 5. *Any HTO equilibrium involves full pooling on a single contract.*

Because a HTO equilibrium is full pooling, the bidders' belief after the offer is posted remains their prior, q_0, and the contract chosen solves

$$\max_{U,P} U + F_H(q_0|P)$$
$$s.t. \quad F_B(q_0|P) - U \geq 0.$$

The constraint ensures that bidders are willing to accept the offer. Clearly, the constraint binds at any solution. In addition, if immediate execution is optimal, then there is no distinction between the contingent and upfront transfers (only $U + P$ matters). Hence it is without loss to set $P = P_0(q_0)$ in this case, reducing the problem to

$$\max_{P \geq P_0(q_0)} F_H(q_0|P) + F_B(q_0|P). \tag{4}$$

The following assumption guarantees that the solution to (4) is unique.

Assumption 2. $\frac{k_H}{V_H} \geq \left(1 - \frac{u}{2}\right)$.

We generalize our results when Assumption 2 does not hold in Proposition 11.

Proposition 10. *Under Assumption 2, the HTO equilibrium is unique. Moreover,*

(i) if $q_0 < b(k_H)$, the price is strictly above k_H and the acquirer conducts due diligence.

(ii) if $q_0 \geq b(k_H)$, there is immediate execution.

Compared to the constrained efficient outcome, the HTO equilibrium features a higher price and too much due diligence. The HTO equilibrium also requires a higher initial belief

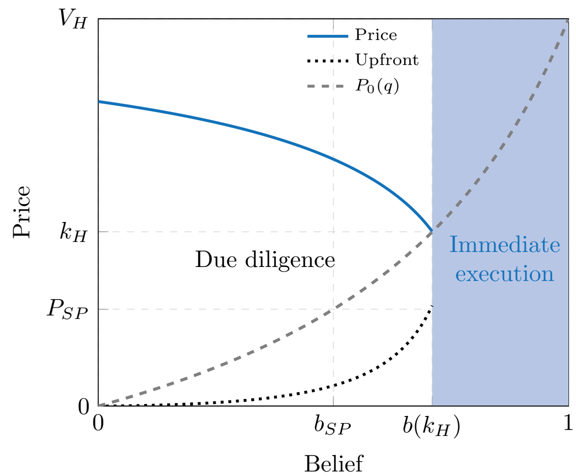


FIGURE 6: Equilibrium Contract with Asymmetric Information. The parameters used to construct this figure satisfy Assumption 2.

in order to forgo due diligence and immediately execute the deal. Intuitively, the greater price benefits the high type, who expects good news and to complete the deal more quickly, at the expense of the low type. Unlike in the baseline model, this inefficiency persists even with the unconditional upfront transfer.

Also in contrast to the baseline model, the high-type optimal price depends on the prior belief as illustrated in Figure 6. Perhaps surprisingly, the high-type optimal price is decreasing in the belief. The intuition is that the high type faces a tradeoff between generating total surplus (which is maximized at $P = P_{SP}$), and extracting surplus from the low type with a higher price. When bidders believe the low type is more likely, extraction becomes relatively more important.

Assumption 2 fails if and only if $k_H < V_H/2$ and γ is sufficiently large, in which case we have the following characterization.

Proposition 11. *If Assumption 2 fails, there exists a pair* $(\overline{q}_H, \underline{q}_H)$, $\overline{q}_H \geq \underline{q}_H > b(k_H) > b_{SP}$, *such that the HTO equilibrium is unique for all* $q_0 \notin (\underline{q}_H, \overline{q}_H)$. *Moreover,*

(i) if $q_0 < \underline{q}_H$, *the price is above* k_H *and the acquirer conducts due diligence.*

(ii) if $q_0 > \overline{q}_H$, *there is immediate execution.*

Under Assumption 2, $\overline{q}_H = \underline{q}_H = b(k_H)$. So, there are two differences without Assumption 2. First, we only have proven that $\overline{q}_H \geq \underline{q}_H$, though in all of the numerical examples we have analyzed, $\overline{q}_H = \underline{q}_H$. Second, and of more economic interest, with a higher quality information process, the high type induces due diligence for a strictly larger set of priors.

4.3 When Due Diligence Information is Contractible

Thus far, we have assumed that the parties cannot contract directly on the information uncovered by due diligence, X, which is modeled as an exogenous stochastic process. In practice, X is potentially manipulable and may be privately observed by the acquirer. Therefore, it is unrealistic to assume that parties can contract directly on X. Nevertheless, the case with contractible X is a useful benchmark. We formally analyze this benchmark in Appendix B and provide a summary of our findings below.

With contractible X and unbounded transfers, it is possible to get arbitrarily close to the first-best outcome, meaning trade occurs almost immediately with the high type and never with the low type, while simultaneously giving all of the surplus to the high-type seller.[21] However, to accomplish this approximation the contingent price must be arbitrarily sensitive to a small amount of information. Moreover, we encounter a Mirrleesian existence problem.

It is not surprising that allowing the execution price to depend on the path of X facilitates a separating equilibrium in which high-value assets trade with probability close to one. What is less obvious is that there exists a separating equilibrium in which this trading occurs almost *immediately*. The construction relies on two new possibilities available with a history contingent price process. First, the execution price depends on the information revealed during due diligence: it is lower following revelation of bad news. This makes the high type's contract relatively less attractive to the low type. However, it also gives incentive to the acquirer to delay execution until enough negative information arrives, which constrains the ability to induce efficient execution. The second feature, which eliminates this incentive, is a non-stationary price process. Once enough good (or bad) information is revealed, the price

[21] In fact, any division of the first-best surplus between the high and low type can be approximated.

is fixed thereafter independent of any information that follows.

5 Additional Considerations and Discussion

In this section, we consider several other extensions of the baseline model. First, we consider the model in which bidding is dynamic and concurrent with learning about the asset, which has different implications when there is common knowledge of gains from trade. Next, we allow for costly due diligence, deal termination, and explore the role of break-up fees. Finally, we explore how debt financing affects the acquirer's incentives and the seller's payoff.

5.1 The Timing of Due Diligence

In practice, some amount of investigation takes place before or during negotiations between bidders and the seller. We show here that our results are unchanged when allowing for due diligence before or during bidding.

The game again consists of two phases. However, the first stage is now dynamic, with competitive bidders conducting due diligence while simultaneously making offers. To facilitate comparison with the baseline model, we assume that all bidders observe the same information process in the first stage (as given in (1)). Let ν be the time at which the seller accepts an offer and denote the offer by P_ν. After the seller accepts an offer, the due diligence subgame ensues: the acquirer can continue to perform due diligence and decide when if ever to execute the transaction at the price P_ν. Applied to corporate takeover auctions, one can interpret bids made prior to ν as indicative bids (Ye, 2007; Quint and Hendricks, 2018; Liu and Officer, 2019) made during the "pre-public" phase of the negotiation (see Section 1.1). Whereas at date ν, the seller solicits formal bids.

The main insight of this section is that allowing for due diligence prior to (or during) bidding does not substantively alter the model's predictions.[22]

[22]This finding is in contrast to Cong (2020), where the seller may optimally choose to (inefficiently) delay the date of the auction in order to reduce privately informed bidders' information rents.

Proposition 12 (Timing Irrelevance)**.** *In any equilibrium with dynamic bidding, the payoffs of the seller and the acquirer are the same as in the equilibrium of the baseline model.*

Recall that due to Assumption 1, the seller's equilibrium payoff in the baseline model is same as in the solution to the hypothetical stopping problem of when to accept $P_0(q_t)$ (see (HSP)). With dynamic bidding, the seller can do no worse: by definition, bidders are willing to offer $P_0(q_t)$ and execute immediately. The interesting part is that the seller can also do no better. This result follows from the seller-optimal price being independent of the prior in the due diligence region. Regardless of what information is uncovered during due diligence, the seller has no desire to renegotiate the price. Hence, she has nothing to gain by waiting to accept an offer.

For $q < b(P^*)$, the price offers and the acceptance time are not uniquely determined with dynamic bidding. In one equilibrium, bidders offer $P_0(q_t)$ at all t and $\nu = \tau(P^*)$, meaning that due diligence is conducted only *prior* to reaching an agreement and never *after* the seller accepts. In another equilibrium, bidders offer $P_S(q_t)$ at all t, $\nu = 0$, and due diligence only takes place after the seller accepts (as we assume in the baseline model). However, in any equilibrium the execution price and execution time are P^* and $\tau(P^*)$ respectively. Hence, both the seller and the acquirer's payoffs are identical.

Assumption 1 is sufficient to ensure timing irrelevance. It is also necessary in the sense that if it fails, then there exists priors such that the seller prefers dynamic bidding. However, rather than explore this relatively small region of the parameter space (see the discussion following Lemma 2), we extend the parameter space to allow for common knowledge of gains from trade (i.e., $k < V_L$), which necessarily violates Assumption 1.

5.2 Common Knowledge of Gains from Trade

Thus far we have assumed that the efficient allocation of the asset is type dependent ($V_L < k < V_H$). If instead $k < V_L$, then there is common knowledge of gains from trade (CKGT). In this case, it is socially optimal to execute the transaction immediately for all prior beliefs. We have already seen the equilibrium without CKGT features excessive due diligence. Under what conditions does a similar result obtain with CGKT? The answer is provided by the following proposition.

Proposition 13 (Equilibrium with CGKT)**.** *Suppose that $k < V_L < V_H$. Then, there exists a $\underline{\gamma} > 0$ such that the equilibrium with static bidding is as follows.*

(i) If $\gamma > \underline{\gamma}$, there exist two thresholds $q_a < q_b$ such that

(a) For $q_0 \in (q_a, q_b)$, the winning offer is $P_0(q_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(q_b))$.

(b) For $q_0 \notin (q_a, q_b)$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

(ii) If $\gamma \leq \underline{\gamma}$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

When the speed of learning relative to the cost of delay is below $\underline{\gamma}$, due diligence is too costly to appeal to the seller. More interestingly, when γ is above $\underline{\gamma}$, due diligence once again emerges. For intermediate beliefs, the seller prefers a price higher than the one that would induce the acquirer to execute immediately. Thus, even when there is no social motive for due diligence, the seller can benefit from inducing the acquirer to conduct due diligence as a means of reducing the acquirer's rents.[23]

One novel implication of the equilibrium structure in Proposition 13 is that dynamic bidding is no longer payoff equivalent to static bidding.

Proposition 14 (Timing Relevance with CKGT)**.** *Suppose that $k < V_L$. Then, there exists a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$, the seller strictly prefers dynamic bidding. Moreover, with dynamic bidding, all due diligence takes place prior to the seller accepting an offer.*

To understand why the seller prefers dynamic bidding, suppose that the initial prior is $q_0 \in (q_a, q_b)$ and thus the winning offer with static bidding is $P_0(q_b)$. If bad news is revealed

[23]When Assumption 1 fails and $k \geq V_L$, the equilibrium with static bidding is similar to Proposition 13, part (i), with the addition of a third belief threshold $q_c \leq q_a$ such that for $q_0 < q_c$, the offer is $P_0(q_c)$ and the acquirer conducts due diligence until $\tau^*(q_c)$.

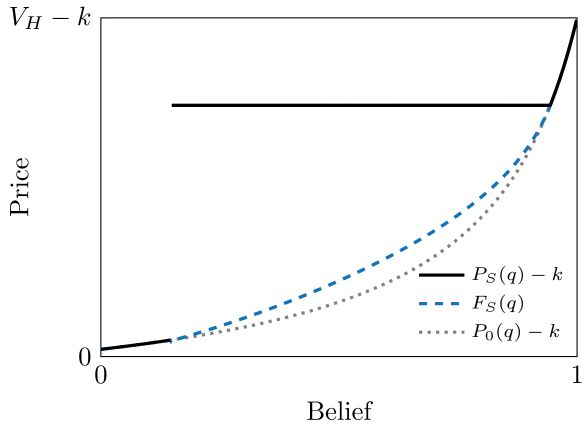


Figure 7: Equilibrium Price and the Seller's Payoff with CKGT

during due diligence such that the belief drops below q_a, then the seller optimal price also drops (see Figure 7). The seller would therefore like to renegotiate the price down to induce the acquirer to execute immediately, which is not feasible under static bidding.[24] However, with dynamic bidding, the seller can accomplish the same outcome by delaying acceptance until due diligence has been effectively completed (i.e., the belief reaches either q_a or q_b) prior to accepting an offer.

Proposition 14 has implications for whether due diligence takes place before or after the seller and acquirer agree to terms. In a strategic acquisition, the primary motivation for the bidder to acquire the target is due to synergies, which corresponds to gains from trade with the target regardless of the value of the firm's existing assets. Whereas a financial bidder is typically only interested in acquiring targets that are undervalued.[25] Our model suggests that in strategic acquisitions, there is an advantage to conducting due diligence prior to terms being reached (e.g., in the "pre-public" phase), whereas there is no such advantage for purely financial transactions.[26] Thus, our results suggest that more of the due diligence will take place in the "pre-public" phase for deals with strategic bidders compared to

[24]Moreover, if the acquirer were allowed to renegotiate, the seller would be subjected to a hold-up problem.

[25]This difference in the motivations of strategic versus financial bidders is discussed in Gorbenko and Malenko (2014).

[26]Of course, providing multiple strategic bidders with access to sensitive information about the firm may impose costs on the seller, non-disclosure agreements notwithstanding.

those with financial bidders. Another prediction relates to the likelihood of deal completion. With dynamic bidding and CKGT, the deal is completed with probability one. A testable implication of this line of argument then is that deals with financial acquirers should be more likely to fail.

5.3 Costly Due Diligence and Break-up Fees

Up to now we have assumed that due diligence does not consume any of the acquirer's resources. In practice, there are costs to performing due diligence. For example, the acquirer typically needs to hire accountants to verify financial statements and/or inspectors to evaluate the condition of physical capital. When due diligence is costly, the acquirer may prefer to terminate the transaction if enough negative information is revealed rather than continue to conduct due diligence. In this section, we extend the baseline model to allow for costly due diligence and deal termination.

Specifically, assume that the acquirer pays a flow cost c until she completes the deal, which happens at τ, or terminates the deal, which happens at ζ. The acquirer chooses a completion and termination time to maximize her payoff

$$F_B^c(q|P) = \sup_{\tau,\zeta} \mathbb{E}_q \left[\mathbb{I}_{\{\tau<\zeta\}} e^{-r\tau}(V(q_\tau) - P) - \int_0^{\min\{\tau,\zeta\}} e^{-rt} c dt \right].$$

Fixing any $P \in (k, V_H)$, the acquirer's optimal strategy is to complete the deal for beliefs above an upper threshold $\hat{b}(P)$, to terminate the deal for beliefs below a lower threshold $\hat{a}(P)$, and to conduct due diligence for beliefs in between.[27] Our main results from the baseline model can be extended to the case with costly due diligence. For any $P > k$, the acquirer's execution threshold remains above the socially efficient execution threshold. In addition, the termination threshold is also above the socially efficient termination threshold. Intuitively, because the acquirer does not capture all of the surplus from a completed deal, she "gives up"

[27] The upper and lower thresholds can be determined by solving a system consisting of an ODE for the acquirer's value function and four boundary conditions (value matching and smooth pasting at each boundary) and four unknowns (two constants in the acquirer's value function and the two boundaries).

prior to when doing so is socially optimal. In that sense, there is both inefficient execution and inefficient termination.

The profit maximizing and socially efficient outcome can be obtained with an upfront transfer and a contingent price $P = k$ (i.e., Proposition 6 still holds). If the acquirer is unable to offer an upfront transfer (e.g., due to financing/liquidity constraints), then a break-up fee paid from the acquirer to the seller conditional on termination can improve the seller's payoff by inducing both earlier execution and later termination. In fact, absent discounting ($r = 0$) there is an equivalence between the use of break-up fees and the use of upfront transfers. Precisely, if $r = 0$ then a contract $C = (U, P)$ is functionally equivalent to a contract with a break-up fee of $B = U$ and a contingent price $P + U$, and the optimal mechanism can be implemented with a price/break-up fee pair.

Corollary 2. *Consider the model with costly due diligence and no discounting (i.e., $r = 0$). The socially optimal and seller optimal mechanism can be implemented with a break-up fee $B = F_B^c(q|k)$ and a contingent price $P = k + B$.*

Chen et al. (2020) document that 21% of M&A deals have a bidder break-up provision with an average break-up fee size of 3.6% of the deal value. Bates and Lemmon (2003) find similar results and argue that break-up fees enable the seller to lock in the surplus of the deal (whether or not it is executed), which is consistent with their role in our model.

5.4 Debt (and Seller) Financing

Many transactions (such as leveraged buyouts and mortgage-financed real estate) rely on debt financing from a third party. We now explore how debt financing impacts the acquirer's incentives during due diligence and the equilibrium outcome. We introduce a risk-neutral financier with deep pockets. Due to competition, the financier must break even in expectation. The acquirer pays the seller using the funds raised from a debt contract with the financier.

Let F denote the face value of the debt contract. The acquirer also has existing assets in place with value $A > 0$ that are pledged to the financier in case of default. We assume that A is small enough that if the transaction is executed and the asset is low value, then the acquirer will default on the debt and the financier receives A.[28] If the transaction is executed and the asset is high value, then the acquirer will repay the face value.

Given a debt contract, the acquirer's problem in the due diligence game is

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau} (V(q_\tau) - D(q_\tau)) \right],$$

where $D(q) = qF + (1-q)A$ is the expected value of the debt payments. As in the baseline model, the solution to the acquirer's problem involves a belief threshold, denoted $b_d(F)$. Therefore, if P is the price paid to the seller (by the financier), then the financier will break even if and only if $P = D(b_d(F))$.

By inverting $b_d(F)$ and substituting into the break even condition, we get $P_D(q) = q b_d^{-1}(q) + (1-q)A$, which is the maximal amount the acquirer can credibly bid to the seller with an execution threshold of q. $P_D(q)$ is the analog of $P_0(q)$ in the baseline model. We say that an execution threshold q is *feasible* if $V(q) \geq k$. Equilibrium conditions dictate that execution cannot take place at an infeasible threshold.

Lemma 6. *Suppose that* $A < \bar{A} \equiv k \frac{u-1}{u-k/V_H}$, *then* $P_D(q) > P_0(q)$ *for any feasible execution threshold.*

An implication of Lemma 6 is that the seller prefers the acquirer to finance the transaction with debt. Intuitively, financing the purchase with debt makes it less costly to acquire a low value asset because the acquirer can default. As a result, the acquirer is willing to execute the transaction more quickly, which the seller also prefers. Because the financier earns zero expected profit, there is, in fact, no need for a third-party to finance the transaction. The seller can (and should, if necessary) serve as the financier in order to achieve the same expected payoff. This prediction is consistent with Jansen (2020), who documents that

[28] If A is sufficiently large, then the acquirer will never default on the debt and the model is equivalent to the baseline model.

seller financing is a common feature of private firm acquisitions, occurring in 50% of the transactions in the sample.

Two additional remarks are worth noting. First, it can be shown that the equilibrium with debt financing involves an execution threshold that is strictly between the social optimum and the equilibrium of the baseline model. Thus, debt financing improves the efficiency of the due diligence process, though there is still too much compared to the social optimum. Second, the condition on A is not needed for the result in Lemma 6 if we enrich the model to allow the fraction of assets pledged to be a choice variable. A natural direction for future work would be to fully endogenize the acquirer's capital structure as well as the securities used to finance the transaction and compensate the seller.

6 Conclusion

Due diligence is common practice prior to the execution of large transactions. We propose a model of the due diligence process and analyze its effect on prices, payoffs, and deal completion. If the acquirer has the right to conduct due diligence and determine if/when to execute, then the asset she acquires when the seller accepts the offer is a real option, which has important economic implications for trading outcomes.

Most notably, we show that the acquirer engages in "too much" due diligence relative to the social optimum and can extract a positive surplus even with perfect competition among potential buyers. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting with no due diligence. The faster the acquirer can learn during due diligence, the higher is the equilibrium price and the more likely the deal is to fail, whereas the expected duration of due diligence may increase. With symmetric information, the optimal mechanism involves both a price contingent on execution as well as a non-contingent transfer, resembling features such as earnest money or termination fees that are commonly observed in transactions involving due diligence. We considered various extensions of our baseline model and discussed implications for empirical work.

References

Banks, J. S. and Sobel, J. (1987). Equilibrium selection in signaling games. *Econometrica*, 55(3):647–661.

Bates, T. W. and Lemmon, M. L. (2003). Breaking up is hard to do? An analysis of termination fee provisions and merger outcomes. *Journal of Financial Economics*, 69(3):469–504.

Betton, S., Eckbo, B. E., and Thorburn, K. S. (2008). Corporate takeovers. In *Handbook of Empirical Corporate Finance*, pages 291–429. Elsevier.

Board, S. (2007). Selling options. *Journal of Economic Theory*, 136(1):324–340.

Boone, A. L. and Mulherin, J. H. (2007). How are firms sold? *The Journal of Finance*, 62(2):847–875.

Brueggeman, W. B. and Fisher, J. D. (2019). *Real estate finance and investments*. McGraw-Hill Education, 16th edition.

Bulow, J. and Klemperer, P. (1996). Auctions versus negotiations. *American Economic Review*, 86(1):180–194.

Chen, Z., Mahmudi, H., Virani, A., and Zhao, X. (2020). Why are bidder termination provisions included in takeovers? *Journal of Financial and Quantitative Analysis, forthcoming*, pages 1–47.

Cho, I. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Cole, R. A., Ferris, K. R., and Melnik, A. (2016). The direct cost of advice in m&a transactions in the financial sector. pages 1–20.

Cong, L. W. (2018). Auctions of real options. pages 1–51.

Cong, L. W. (2020). Timing of auctions of real options. *Management Science*, 66(9):3956–3976.

Courty, P. and Hao, L. (2000). Sequential screening. *The Review of Economic Studies*, 67(4):697–717.

Daley, B. and Green, B. (2012). Waiting for news in the market for lemons. *Econometrica*, 80(4):1433–1504.

Daley, B. and Green, B. S. (2020). Bargaining and news. *American Economic Review*, 110(2):428–474.

Davis, S., Gerstner, E., and Hagerty, M. (1995). Money back guarantees in retailing: Matching products to consumer tastes. *Journal of Retailing*, 71(1):7–22.

DeMarzo, P. M., Kremer, I., and Skrzypacz, A. (2005). Bidding with securities: Auctions and security design. *American Economic Review*, 95(4):936–959.

Dixit, A. K., Dixit, R. K., and Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton university press.

Eckbo, B. E., Malenko, A., and Thorburn, Karin S., S. (2019). Strategic decisions in takeover auctions. pages 1–59.

Fudenberg, D. and Tirole, J. (1991). *Game theory*. MIT press.

Gorbenko, A. S. and Malenko, A. (2011). Competition among sellers in securities auctions. *American Economic Review*, 101(5):1806–1841.

Gorbenko, A. S. and Malenko, A. (2014). Strategic and financial bidders in takeover auctions. *The Journal of Finance*, 69(6):2513–2555.

Gorbenko, A. S. and Malenko, A. (2018). The timing and method of payment in mergers when acquirers are financially constrained. *The Review of Financial Studies*, 31(10):3937–3978.

Gorbenko, A. S. and Malenko, A. (2019). Auctions with endogenous initiation. pages 1–68.

Hansen, R. (2001). Auctions of companies. *Economic Inquiry*, 39(1):30–43.

Harrison, M. J. (2013). *Brownnian model of performance and control*. Cambridge University press.

Heath, D. and Mitchell, M. L. (2020). Market crashes and merger completions. pages 1–55.

Inderst, R. and Ottaviani, M. (2013). Sales talk, cancellation terms and the role of consumer protection. *The Review of Economic Studies*, 80(3):1002–1026.

Jansen, M. (2020). Resolving information asymmetry through contractual risk sharing: The case of private firm acquisitions. *Journal of Accounting Research*, 58(5):1203–1248.

Kohlberg, E. and Mertens, J.-F. (1986). On the strategic stability of equilibria. *Econometrica*, 54(5):1003–1037.

Krähmer, D. and Strausz, R. (2015). Optimal sales contracts with withdrawal rights. *The Review of Economic Studies*, 82(2):762–790.

Lajoux, A. R. (2010). *Art of M and A Due Diligence*. McGraw-Hill Professional.

Liu, T. and Officer, M. S. (2019). Inside the 'black box' of private merger negotiations. pages 1–66.

Mailath, G. J., Okuno-Fujiwara, M., and Postlewaite, A. (1993). Belief-based refinements in signalling games. *Journal of Economic Theory*, 60(2):241–276.

Marquardt, C. and Zur, E. (2015). The role of accounting quality in the M&A market. *Management Science*, 61(3):604–623.

Matthews, S. (1984). Information acquisition in discriminatory auctions. In *Bayesian Models in Economic Theory*, pages 181–207. Elsevier.

Matthews, S. A. and Persico, N. (2007). Information acquisition and refunds for returns. pages 1–35.

Offenberg, D. and Pirinsky, C. (2015). How do acquirers choose between mergers and tender offers? *Journal of Financial Economics*, 116(2):331–348.

Persico, N. (2000). Information acquisition in auctions. *Econometrica*, 68(1):135–148.

Quint, D. and Hendricks, K. (2018). A theory of indicative bidding. *American Economic Journal: Microeconomics*, 10(2):118–51.

Shi, X. (2012). Optimal auctions with information acquisition. *Games and Economic Behavior*, 74(2):666–686.

Snow, B. (2011). *Mergers and acquisitions for dummies*. John Wiley & Sons.

Stegeman, M. (1996). Participation costs and efficient auctions. *Journal of Economic Theory*, 71(1):228–259.

Wangerin, D. (2019). M&A due diligence, post-acquisition performance, and financial reporting for business combinations. *Contemporary Accounting Research*, 36(4):2344–2378.

Ye, L. (2007). Indicative bidding and a theory of two-stage auctions. *Games and Economic Behavior*, 58(1):181–207.

A Appendix

In the appendix, we will sometimes use beliefs transformed into a log-likelihood ratio, denoted $z \equiv \ln\left(\frac{q}{1-q}\right)$. The dynamics of the log-likelihood of the belief process, $Z_t = \ln\left(\frac{q_t}{1-q_t}\right)$, are given by

$$dZ_t = \frac{\phi^2}{2}\left(2p(Z_t) - 1\right)dt + \phi dB_t,$$

where $p(z) = \frac{e^z}{1+e^z}$, which transforms a likelihood ratio to a probability. We use $\beta(P)$ to denote the log-likelihood ratio of $b(P)$.

Let $\mathcal{A}$ denote the infinitesimal generator of Z, which is defined to act on suitable functions f by $\mathcal{A}f(z) = \lim_{t\downarrow 0}\frac{\mathbb{E}_z[f(Z_t)]-f(z)}{t} = \frac{\phi^2}{2}\left((2p(z)-1)f' + f''\right)$.

A.1 Proofs for Section 2

Proof of Lemma 1. We will construct the value function $F_B(z|P)$ given the candidate strategy $\tau^*(P) = \inf\{t > 0 | q_t \geq b(P)\}$ and show that it solves the Hamilton-Jacobi-Bellman equation

$$0 = \max\left\{-rF_B(z|P) + \mathcal{A}F_B(z|P), (\mathbb{E}_z[V_\theta] - P) - F_B(z|P)\right\},$$

and is smooth (C^1, C^2 a.e.), which is sufficient to ensure that $\tau^*(P)$ solves the acquirer's problem (Harrison, 2013, Theorem 5.1).

Given the candidate $\tau^*(P)$, $F_B(z|P) = \mathbb{E}_z[V_\theta] - P$ for $z \geq \beta(P) \equiv \ln\left(\frac{b(P)}{1-b(P)}\right)$, and F_B satisfies $rF_B(z|P) = \mathcal{A}F_B(z|P)$ for $z < \beta(P)$. Thus, there are two things to check in order to verify F_B satisfies the HJB: (1) that $-rF_B(z|P) + \mathcal{A}F_B(z|P) \leq 0$ for $z > \beta(P)$ and (2) that $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$ for $z < \beta$.

For (1), given $F_B(z|P) = \mathbb{E}_z[V_\theta] - P$, we can compute

$$-rF_B(z|P) + \mathcal{A}F_B(z|P) = \frac{r(P + e^z(P - V_H) - V_L)}{1 + e^z}.$$

The RHS has the same sign as $P + e^z(P - V_H) - V_L$, which is decreasing in z (since $P < V_H$) and at $\beta(P)$ is equal to

$$P + e^{\beta(P)}(P - V_H) - V_L = \frac{-P + V_L}{u - 1} < 0,$$

which verifies (1). For (2), for $z < \beta(P)$, $F_B(z|P)$ solves $rF_B(z|P) = \mathcal{A}F_B(z|P)$. It is therefore of the form

$$C_1 \frac{e^{uz}}{1+e^z} + C_2 \frac{e^{\hat{u}z}}{1+e^z}$$

with $(u, \hat{u}) = \frac{1 \pm \sqrt{1+\frac{8r}{\phi^2}}}{2}$. Furthermore, F_B is bounded and satisfies the boundary condition

$$F_B(\beta(P)|P) = \mathbb{E}_z[V_\theta] - P$$

which imply that

$$C_1 = e^{-\beta(P)u} \left(-\left(1 + e^{\beta(P)}\right) P + e^{\beta(P)} V_H + V_L \right),$$
$$C_2 = 0.$$

To see that $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$, define

$$G(z) = \frac{\partial \left(F_B(z|P) - (\mathbb{E}_z[V_\theta] - P) \right)}{\partial z}.$$

By construction, $G(\beta(P)) = 0$. Furthermore, $G(z)$ has the same sign as

$$\frac{(1+e^z)^2}{e^z} G(z)$$

Differentiating this function with respect to z gives

$$\frac{\partial \left(\frac{(1+e^z)^2}{e^z} G(z) \right)}{\partial z} = u \left(e^z + 1\right) (P - V_L) e^{(u-1)z} \left(\frac{u(V_L - P)}{(u-1)(P - V_H)} \right)^{-u} > 0,$$

which shows that $G(z) < 0$, and therefore $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$, for $z < \beta(P)$.

Finally, $F_B(z|P)$ is C^2 everywhere except at $z = \beta(P)$ by construction and it is C^1 at $z = \beta(P)$ since $\beta(P)$ follows from the smooth pasting condition $F'_B(\beta(P)^-|P) = F'_B(\beta(P)^+|P)$. Therefore, we can apply Theorem 5.1 of Harrison (2013), which proves that $\tau^*(P)$ is the optimal stopping time. □

In the proof above, we allowed V_L to be arbitrary because this proof will also be used when we study the model with common knowledge of gains from trade (Section 5.2) where $V_L > k$. From now until Section A.4, we normalize $V_L = 0$ without loss.

Consider the stopping problem (HSP). Let $h(t, q_t) = e^{-rt}(P_0(q_t) - k)$ from stopping at time t. By Dynkin's formula, the payoff under an arbitrary stopping time with finite expectation is

$$\mathbb{E}_q[h(\tau, q_\tau)] = h(0, q) + \mathbb{E}_q\left[\int_0^\tau \hat{\mathcal{A}}h(s, q_s)ds\right]$$

where $\hat{\mathcal{A}}$ is the characteristic operator of the process (t, q_t) under $\mathcal{P}$. That is,

$$\hat{\mathcal{A}}h(q,t) = re^{-rt}\left(\frac{\gamma}{2}q^2(1-q)^2 P_0''(q) - (P_0(q) - k)\right). \tag{5}$$

Lemma A.1. *There exists a $\bar{k} \in (0, V_H)$ such that $\hat{\mathcal{A}}h$ satisfies single crossing with respect to q for all $k \geq \bar{k}$.*

Proof. The expression inside the parentheses of (5) is independent of t. Thus, it suffices to show that it this expression satisfies single crossing w.r.t. q for all $k \geq \bar{k}$. The first term of the expression,

$$\frac{\gamma}{2}q(1-q)P_0''(q) = -\frac{2(q-1)^2q^2V_H}{(q-u)^3},$$

is weakly positive and equal to zero at $q = 1$. Furthermore, $P_0(q)$ is strictly increasing in q. Hence, the expression attains its minimum at $q = 1$, which is $-(V_H - k) < 0$. Further, the expression is increasing in k and equal to k at $q = 0$. To characterize $\bar{k}$, note that

$$\frac{\gamma}{2}q(1-q)P_0''(q) - P_0(q)$$

is a rational function in q and therefore has a finite number of local minima, denoted $\{q_1, q_2, ..., q_n\}$, on $(0, 1)$. Let $k_i = P_0(q_i) - \frac{\gamma}{2}q_i(1-q_i)P_0''(q_i) < V_H$ and define

$$\bar{k} = \max\{0, k_1, k_2, ..., k_n\}.$$

For all $k \geq \bar{k}$, all local minima of the expression are weakly positive, while the global minimum (attained at $q = 1$) is negative. Hence, for all such k, the expression crosses zero exactly once and from above, as desired. □

Lemma A.2. *There exists a $\bar{\gamma}$ such that $\hat{\mathcal{A}}h$ satisfies single crossing with respect to q for all $\gamma > \bar{\gamma}$.*

Proof. The constant $u > 1$ is decreasing in γ since $u = \frac{1+\sqrt{\frac{8}{\gamma}+1}}{2} > 1$. Define

$$g(q) = -(q-u)^3\frac{e^{rt}}{r}\hat{\mathcal{A}}h,$$

which has the same sign has $\hat{\mathcal{A}}h$, since $u > 1$ for any $\gamma \in [0, \infty)$. Hence, it suffices to demonstrate that g satisfies single crossing for $\gamma \geq \bar{\gamma}$.

Observe that $g(0) = u^3k > 0$, $g(1) = (u-1)^3(k-V_H) < 0$, and $g'(0)-3ku^2-(u-1)u^2V_H < 0$. Furthermore, $g''(q)$ is quadratic in q and $g''(0) = 6ku + 4(1 + (-1 + u)u)V_H > 0$. Finally, $g''(1) = 6k(-1 + u) + 2(5 + u(-5 + 2u))V_H$, which is positive for u sufficiently close to 1. Define $\bar{\gamma}_1 = \inf\{\gamma | g''(1) > 0\}$, and assume from now on that $\gamma > \bar{\gamma}_1$, which implies that $g''(q)$ changes sign either never or twice on $q \in [0, 1]$. We break the remainder of the proof into two cases:

Case 1: $k/V_H > 1/4$. For u sufficiently close to one, using the fact that $\frac{k}{V_H} > \frac{1}{4}$, we get that

$$g'''(q) = -6(k + (3 - 8q + u)V_H) < 0.$$

Define

$$\bar{\gamma}_2 = \inf\{\gamma | g'''(q) < 0 \ \forall q \in [0, 1]\}$$

and assume from now on that $\gamma > \max_{i\in\{1,2\}} \bar{\gamma}_i$. As a result $g'' > 0$, which implies that g' crosses zero at most once (it is first negative and then positive or always negative) and as a result g crosses zero once from above, as desired.

Case 2: $k/V_H \leq 1/4$ We already established that $g''(0) > 0$ and $g''(1) > 0$. For $u = 1$, the largest solution to $g''(q) = 0$ is given by

$$\frac{3k + 12V_H + \sqrt{9k^2 - 72kV_H + 48V_H^2}}{24V_H} < \frac{3 + 12 + \sqrt{48}}{24} < 1.$$

This implies that for u sufficiently close to one, there exists a $\hat{q} < 1$ such that $g''(q) > 0$ for $q > \hat{q}$. Define

$$\bar{\gamma}_3 = \inf\left\{\gamma \middle| \exists\ q < \frac{3 + 12 + \sqrt{48}}{24} \ \textit{such that}\ g''(q) = 0\right\}.$$

and assume from now on that $\gamma > \max_{i\in\{1,2,3\}} \bar{\gamma}_i$.

Therefore, for $q > \hat{q}$ we have that $g'(q)$ crosses zero at most once and if it does then it crosses from below. This implies that for $q > \hat{q}$ $g(q)$ crosses zero at most once.

For $q \in [0, \hat{q})$ and for u sufficiently close to one the operator is positive because for $u = 1$ the operator becomes

$$\lim_{u\to 1} \hat{\mathcal{A}}h(q, t) = e^{-rt}r\left(k + \frac{2q^2V_H}{1 - q}\right) > 0.$$

Define

$$\bar{\gamma}_4 = \inf \left\{ \gamma \middle| g(q) > 0 \forall q \in \left[0, \frac{3 + 12 + \sqrt{48}}{24}\right] \right\}.$$

and let $\bar{\gamma} \equiv \max_{i \in \{1,2,3,4\}} \bar{\gamma}_i$. The last two steps imply that for $\gamma \geq \bar{\gamma}$, $\hat{\mathcal{A}}h$ satisfies single crossing. ☐

Proof of Lemma 2. Lemma A.1 and A.2 imply that $\hat{\mathcal{A}}h$ satisfies single crossing and therefore the optimal stopping strategy for the accompanying stopping problem is a threshold strategy, see (Dixit et al., 1994, Ch. 5). ☐

Proof of Lemma 3. Clearly, (HSP) provides an upper bound on the seller's equilibrium payoff. We claim that $P_S(q)$ (uniquely) achieves this bound. To verify the claim, note that $b(P^*) = b(P_0(q^*)) = q^*$. For $q > q^*$, the claim is immediate since both involve immediate execution at $P_0(q)$. For $q \leq q^*$, the acquirer's stopping rule in the due diligence game is τ_{hyp} and the seller's payoff is $E_q[e^{-r\tau_{hyp}}(P_0(q^*) - k)]$, which is exactly the same as the solution to (HSP) given Assumption 1. Moreover, any price different other than $P_S(q)$ will lead to a strictly different (and thus lower) payoff for the seller than obtained in the solution to (HSP). ☐

Proof of Proposition 1. From Lemma 1 it follows that the threshold stopping rule $b(P)$ is aquirer-optimal for any price P. Given this stopping rule, Lemma 3 shows that the seller's payoff is uniquely maximized by the price offer

$$P_S(z) = \begin{cases} P^* & \textit{if } q \leq b(P^*) \\ P_0(q) & \textit{if } q > b(P^*). \end{cases}$$

By standard arguments, due to competition among bidders, the winning offer must maximize the seller's payoff. ☐

Proof of Lemma 4. The statement in (i) follows from the martingale property of q_t: as $t \to \infty$ either q_t reaches $b(P^*)$ and the deal is completed, or q_t converges to zero and the deal is never completed. Since $\mathbb{E}_{q_0}[q_t] = q_0 = \mathbb{P}(\tau^* < \infty)b(P^*)$, we have $\mathbb{P}(\tau^* < \infty) = \frac{q_0}{b(P^*)}$.

For (ii), we need to calculate $\mathbb{E}_z[\tau^* | \tau^* < \infty]$. Observe that conditional on θ, Z_t^θ is a Brownian motion with drift $\pm\frac{\phi^2}{2}$ and volatility ϕ. Using Harrison (2013, p. 14), we can then explicitly calculate

$$\mathbb{P}_z\left(\tau^* \le t|\theta = H\right) = 1 - N\left(\frac{(\beta(P^*) - z) - \frac{\phi}{2}t}{\phi\sqrt{t}}\right) + e^{\beta(P^*)-z}N\left(\frac{-(\beta(P^*) - z) - \frac{\phi}{2}t}{\phi\sqrt{t}}\right),$$

$$\mathbb{P}_z\left(\tau^* \le t|\theta = L\right) = 1 - N\left(\frac{(\beta(P^*) - z) + \frac{\phi}{2}t}{\phi\sqrt{t}}\right) + e^{-(\beta(P^*)-z)}N\left(\frac{-(\beta(P^*) - z) + \frac{\phi}{2}t}{\phi\sqrt{t}}\right),$$

where $N(\cdot)$ is the cumulative distribution function of a standard normal random variable. Using these conditional probabilities, we can obtain the cumulative distribution function of τ^* conditional on completion

$$\mathbb{P}_z\left(\tau^* \le t|\tau^* < \infty\right) = \frac{p(z)\mathbb{P}_z\left(\tau^* \le t|\theta = H\right) + (1 - p(z))\,\mathbb{P}_z\left(\tau^* \le t|\theta = L\right)}{\frac{p(z)}{p(\beta(P^*))}}.$$

Differentiating this cumulative distribution function with respect to t yields the probability distribution function

$$\mathbb{P}_z\left(\tau^* = t|\tau^* < \infty\right) = \frac{(\beta(P^*) - z)e^{-\frac{\left(t\phi^2 - 2(\beta(P^*)-z)\right)^2}{8t\phi^2}}}{\sqrt{2\pi}t^{3/2}\phi},$$

which allows us to calculate

$$\mathbb{E}_z\left[\tau^*|\tau^* < \infty\right] = \int_0^\infty t\mathbb{P}_z\left(\tau^* = t|\tau^* < \infty\right)dt = \frac{2}{\phi^2}(\beta(P^*) - z). \qquad \square$$

Proof of Proposition 2. For statement (i), given a price P, the seller's payoff for $z < \beta(P)$ is given by

$$F_S(z|P) = \frac{(P-k)e^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}\left(\frac{Pu}{(u-1)(V_H-P)} + 1\right)}{e^z + 1}.$$

To maximize the seller's payoff with respect to P, first derive

$$\frac{\partial F_S(z|P)}{\partial P} = -\frac{e^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}\left((u-1)V_H^2(P(u-1) - ku) + P^2(u-2)V_H + P^3\right)}{P(u-1)\left(e^z+1\right)\left(P - V_H\right)^2},$$

and note that because $P \in (k, V_H)$, $\partial F_S(z|P)/\partial P$ has the same sign as

$$w(P) \equiv -\left((u-1)V_H^2(P(u-1)-ku)+P^2(u-2)V_H+P^3\right).$$

The first-order condition is therefore $w(P) = 0$, and we can normalize $V_H = 1$ without loss of generality. Because $w(1) < 0$ and w is continuous in P, the largest $P \in [0, 1]$ that solves $w(P) = 0$ is a local maximum of F_S. Therefore, P^* must be this largest solution to $w(P) = 0$. To see why, suppose the largest solution to the first-order condition is not P^*. Then there is a $\tilde{P} > P^*$ which is a local maximum. But then, for $\epsilon > 0$ small enough, starting from belief $z \in (\beta(\tilde{P}) - \epsilon, \beta(\tilde{P}))$, it is no longer seller payoff-maximizing to execute immediately, which violates Lemma 3.

We next establish that P^* is increasing in ϕ once ϕ is sufficiently large. To start, see that

$$w(\sqrt{k}) = -\left(k-\sqrt{k}\right)\left(\sqrt{k}-(u-1)^2\right),$$

which is strictly positive for ϕ sufficiently large (u sufficiently close to one) in which case we must have $P^* > \sqrt{k}$. Taking the derivative of w with respect to u evaluated at P^* yields

$$\left.\frac{\partial w(P)}{\partial u}\right|_{P=P^*} = k(-1+2u) - P^*(-2+P^*+2u),$$

which is negative for ϕ sufficiently large since in that case $P^* > \sqrt{k}$ and u is close to 1. Therefore, given P^* a small increase in ϕ will make $w(P) > 0$ and as a result P^* is increasing in ϕ (for ϕ sufficiently large).

Finally, we show that P^* is increasing for all ϕ. To start, see that P^* is continuous in ϕ because $w(P)$ is continuous in ϕ and $w(k) > 0 > w(1)$ for any ϕ. Now, suppose that P^* is decreasing in ϕ for some ϕ. Then there exists some price $\hat{P}^* < 1$ such that there exist two $u_1, u_2 > 1$ for which $w(\hat{P}^*|u_1) = w(\hat{P}^*|u_2) = 0$. Observe that for any $P < 1$, $w(P|u)$ is quadratic in u and therefore can have at most two roots. Further, for any $P < 1$, $w(P|u = 1) = P^2(1 - P) > 0$. Since, the coefficient on u^2 in $w(P|u)$ is $k - P < 0$ (for all candidate equilibrium prices), $w(\hat{P}^*|u)$ can have only one root on $u \in [1, \infty)$, which contradicts the implication of our supposition. Therefore, the price P^* must be increasing in ϕ everywhere.

Statement (ii) follows immediately from (i), Lemma 4(i), and $b(P)$ increasing in ϕ. As a consequence, $b(P^*)$ is increasing in ϕ and $\frac{q_0}{b(P^*)}$ is decreasing in ϕ.

For statement (iii), using Lemma 4(ii), derive

$$\frac{\partial \mathbb{E}_z\left[\tau|\tau<\infty\right]}{\partial \phi} = \frac{-4(\beta(P^*)-z)+2\phi\frac{\partial\beta(P^*)}{\partial\phi}}{\phi^3}. \tag{6}$$

Because $b(P^*)$ is increasing in ϕ, (6) is negative for $z < z' = \beta(P^*) - \frac{\phi}{2}\frac{\partial\beta(P^*)}{\partial\phi}$ and positive for $z \in (z', \beta(P^*))$. □

Proof of Proposition 3. Part (iii) of the proposition is immediate since both cases involve immediate execution. We demonstrate (i) and (ii) in three steps. Let $z^* \equiv \ln\left(\frac{q^*}{1-q^*}\right)$. Define $\underline{z} < b(0)$ as the solution to $\mathbb{E}_{\underline{z}}\left[V_\theta - k\right] = 0$.
Step 1: Social surplus is higher with due diligence for $z < \hat{z}$. For $z \leq \underline{z}$, social surplus is strictly positive with due diligence and equal to zero without due diligence:

$$F_S(z|P_S(z)) + F_B(z|P_S(z)) = \mathbb{E}_z\left[e^{-r\tau(P^*)}(V(q^*) - k)\right] > 0 = \max\left\{\mathbb{E}_z\left[V_\theta - k\right], 0\right\}.$$

Both with and without due diligence, surplus is continuous in z. Hence, there exists a $\hat{z} > \underline{z}$ such that this inequality continues to holds for all $z \leq \hat{z}$.
Step 2: Social surplus is lower with due diligence for $z \in (\hat{\hat{z}}, z^*)$. From Proposition 4, the socially optimal execution threshold, denoted $\beta(k)$, is below z^*. Therefore, the outcome without due diligence achieves maximal social surplus on $(\beta(k), z^*)$, while the outcome with due diligence does not. By continuity, there is exists a $\hat{\hat{z}}$ such that the inequality holds on $(\hat{\hat{z}}, z^*)$.
Step 3: $\hat{z} = \hat{\hat{z}}$. For $z \in (\underline{z}, z^*)$, we can differentiate the difference in social surplus with respect to z

$$\frac{\partial\left(\mathbb{E}_z\left[V_\theta - k\right] - \left(F_S(z|P^*) + F_B(z|P^*)\right)\right)}{\partial z}.$$

This derivative has the same sign as

$$\begin{aligned}&\frac{(1+e^z)^2}{e^z}\frac{\partial\left(\mathbb{E}_z\left[V_\theta - k\right] - F_S(z|P^*) - F_B(z|P^*)\right)}{\partial z}\\ =&\frac{e^{(u-1)z}\left((u-1)e^z + u\right)\left(\frac{P^*u}{(u-1)(V_H-P^*)}\right)^{-u}\left(P^*uV_H - k(P^* + (u-1)V_H)\right)}{(u-1)(P^* - V_H)} + V_H,\end{aligned}$$

which is monotone in z and therefore crosses zero at most once. Since the derivative of the difference changes sign at most once on $(\underline{z}, z^*)$, the difference in social surplus crosses zero at most once on $(\underline{z}, z^*)$. Hence, $\hat{z} = \hat{\hat{z}}$. Claims (i) and (ii) of the Proposition have thus been established, where $\bar{q} = p(\hat{z})$. □

Proof of Proposition 4. The proof of the first statement is analogous to the proof of Lemma 1 after replacing P with k. That $k < P^*$ follow from the fact that if $P = k$ then the seller's payoff is zero whereas for any $P > k$, the seller's payoff is strictly positive. Hence, $P^* > k$.

For the second statement, note that the process $q = \{q_t : 0 \leq t \leq \infty\}$ is a bounded martingale. Given an arbitrary execution threshold, b, let $H(q_t) = \mathbb{P}_{q_t}(q_T = b)$. Using the

Kolmogorov Backward Equation, we have that

$$H(q_t) = \mathbb{E}_t[H(q_{t+dt})] = \mathbb{E}\left[H(q_t) + H'(q_t)dq_t + \frac{1}{2}H''(q_t)(dq_t)^2\right],$$

which using the Martingale property, $\mathbb{E}[dq_t] = 0$, implies that $H''(q_t) = 0$. Employing the boundary conditions $H(b) = 1$ and $H(0) = 0$, we can conclude that $H(q) = q/b$ for $q \leq b$. The second statement of the proposition then follows from the fact that $b(P^*) > b(k)$. □

A.2 Proofs for Section 3

Proof of Proposition 5. Take any $q_0 < b(k)$. It is socially optimal to execute the deal the first time beliefs q_t reach $b(k)$, see Proposition 4. Take any deadline $T \in (0, \infty)$ and price P. Let $\tau(P, T)$ be the time at which the acquirer completes the deal and if it does not complete the deal before the deadline then $\tau(P, T) = T$. Because q_t is a diffusion process without a drift

$$\mathbb{P}_0\left(\sup_{t\in[0,\tau(P,T)]} q_t \geq b(k)\right) \leq \mathbb{P}_0\left(\sup_{t\in[0,T]} q_t \geq b(k)\right) < 1.$$

This result tells us that the deal isn't completed socially efficiently with certainty before the deadline. Furthermore, if the deal isn't completed before the deadline a fraction $(1-\alpha)$ of the value is lost and therefore social efficiency cannot be achieved. Finally, we already know that for $T = \infty$ the acquirer is too diligent, see Proposition 4.

For $q_0 \geq b(k)$, the socially efficient outcome is to execute directly but given that the deadlines are positive $T > 0$, the acquirer always extracts some of the surplus. □

Proof of Proposition 6. Clearly, the candidate is an equilibrium. Since due diligence is efficient and the seller extracts all of the surplus, any other offer will either (i) be less attractive to the seller and therefore rejected, or (ii) earn negative expected payoff for the acquirer. In either case, such an offer does not constitute a profitable deviation for bidders. For uniqueness, note that with upfront payments, there is now transferable utility and thus by the standard argument for Bertrand competition, bidders must earn zero profit (otherwise they can increase U by ϵ and win with probability one). Second, for any other candidate equilibrium, there exists an offer that the seller prefers to accept and earns a strictly positive expected profit. Thus, a profitable deviation exists. □

A.3 Proofs for Section 4

Proof of Proposition 7. Recall that in the due diligence subgame the constrained efficient outcome is achieved if and only if $P = P_{SP}$. The payoffs to the acquirer and the seller under a contract (U, P_{SP}) are $F_B(q_0|P_{SP}) - U$ and $F_\theta(q_0|P_{SP}) + U$, respectively. Notice that $F_B(q_0|P_{SP}) \geq 0$ because the acquirer has the right not to execute, and $F_L(q_0|P_{SP}) \geq 0$ because $P_{SP} > k_L$. Hence, there exits a $U \geq 0$ such that (U, P_{SP}) achieves the constrained efficient outcome and satisfies bidder and (both) seller participation constraints if and only if $F_H(q_0|P_{SP}) + F_B(q_0|P_{SP}) \geq 0$. The remainder of the proof establishes that this inequality is equivalent to (3).

First, we need to show that there exists a $\hat{q}$ such that $F(q|P_{SP}) \equiv F_H(q|P_{SP}) + F_B(q|P_{SP}) > 0$ if and only if $q > \hat{q}$. The if part is immediate: since $b_{SP} < 1$, for q close enough to 1, the transaction is executed immediately and $F(q|P_{SP}) = V(q) - k_H \approx V_H - k > 0$.

To see that F is negative for low q, assume that $q < b_{SP}$, then

$$
\begin{aligned}
&F(q|P_{SP}) \\
&= \frac{(q-1)(k_H - V_H)\left(\frac{1}{1-q}\right)^u q^{u-1}\left(\frac{u(k_H V_L - k_L V_H)}{(u-1)(k_H-V_H)(V_H-V_L)}\right)^{1-u}(-k_H u + k_L u + q(V_H - V_L))}{u(-k_H + k_L + V_H - V_L)} \\
&= (q-1)\left(\frac{1}{1-q}\right)^u q^{u-1} m(q)
\end{aligned}
$$

with $\lim_{q\to 0} m(q) < 0$. Therefore, there exists a $\tilde{q} > 0$ such that for $q \in (0, \tilde{q})$ $F(q|P_{SP})$ is negative.

For $q < b_{SP}$, $F(q|P_{SP})$ can cross zero only once, namely when $m(q) = 0$ which has a unique solution. For $q \geq b_{SP}$, $F(q|P_{SP}) = V(q) - k_H$ can also only cross zero once. Furthermore, $F(q|P_{SP})$ is continuous for all q. Hence, $F(q|P_{SP})$ can cross zero at most twice.

Continuity, in combination with the fact that F is negative for sufficiently low q and positive for sufficiently high q, implies that $F(q|P_{SP})$ must cross zero an odd number of times. Thus, $F(q|P_{SP})$ crosses zero exactly once and from below: there exists a $\hat{q} \in (0,1)$ such that $F(q|P_{SP}) > 0$ if and only if $q > \hat{q}$.

There are two possible cases for $\hat{q}$ now:

1. If $\hat{q} \geq b_{SP}$, then $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$ and

   $$\frac{k_H - V_L}{V_H - V_L} \geq b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$$

   which is the case if and only if

   $$u \geq \frac{k_H - V_L}{k_H - k_L}$$

   since b_{SP} is decreasing in u (less informative due diligence means sooner execution).

2. If $\hat{q} < b_{SP}$, then $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$ and

$$\frac{u(k_H - k_L)}{V_H - V_L} < b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}. \tag{7}$$

Observe that

$$k_H(u-1) - u(k_L + V_H - V_L) + V_H = (k_H - V_H)(u-1) - u(k_L - V_L) < 0.$$

Therefore, (7) holds if and only if

$$\frac{u(k_H - k_L)}{V_H - V_L}(k_H(-1+u) + V_H - u(k_L + V_H - V_L)) - u(V_L - k_L) > 0.$$

which is a quadratic equation in u with as solution $u = 0$ and $u = \frac{k_H - V_L}{k_H - k_L}$. Furthermore, the terms in front of u^2 are negative which implies that for $u \in \left(0, \frac{k_H - V_L}{k_H - k_L}\right)$, we have $\frac{u(k_H - k_L)}{V_H - V_L} < b_{SP}$. As a result for $u < \frac{k_H - V_L}{k_H - k_L}$ we must have that $\hat{q} < b_{SP}$.

To conclude, we know that if $u < \frac{k_H - V_L}{k_H - k_L}$ holds then $\hat{q} < b_{SP}$ and $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$ while if it fails then $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$. This is equivalent to saying $\hat{q} = \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{k_H - k_L}\right\}$. □

Proofs for Section 4.2

Taking as given the acquirer's solution to the due diligence subgame, and resulting value functions F_B, F_H, F_L, we consider the first-stage signaling game. Let the seller's utility from posting a contract $C = (U, P)$, which results in a belief $\tilde{q}(C)$ be $u_\theta(C, \tilde{q}(C))$. If the contract is accepted, $u_\theta(C, \tilde{q}(C)) = U + F_\theta(\tilde{q}(C)|P)$; if it is rejected $u_\theta(C, \tilde{q}(C)) = 0$. A posted contract will be accepted if and only if $U \leq F_B(\tilde{q}(C)|P)$. Let $\mathbb{S}_\theta$ be the support of the type-θ seller's strategy.

We use σ to denote an arbitrary strategy profile accompanied by the on-path beliefs uniquely determined via Bayes rule, and in a slight abuse of notation, $u_\theta(\sigma)$ to be the payoff of the type-θ seller in σ. We say that σ is an equilibrium if there exist off-path beliefs that support σ by (i) creating no incentives for any player to deviate and (ii) satisfying the requirements for PBE (Fudenberg and Tirole, 1991, pp. 331-333).

Lemma A.3. *In any PBE σ: (i) if $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, then σ is trivial; and (ii) $u_L(\sigma) = 0$ if and only if σ is trivial.*

Proof. For (i), fix a PBE σ such that there exists $C \in \mathbb{S}_L$, but $C \notin \mathbb{S}_H$. Then $\tilde{q}(C) = 0$. Hence, on the path, if C is posted, the deal will never be executed. This is because there are negative gains from trade when $\theta = L$: if there were positive probability of trade, then

$F_L(0|P) + F_B(0|P) < 0$, and (since U is just a transfer) at least one player earns a negative payoff and would do better to deviate. So, $u_L(\sigma) = 0$, and σ is trivial by (ii).

For (ii), it is immediate that if σ is trivial, then $u_L(\sigma) = 0$. Now suppose that σ is nontrivial. Then there is positive probability of deal execution when $\theta = H$. So there is some $C' = (U', P') \in \mathbb{S}_H$ that is accepted. Hence,

$$u_H(\sigma) = u_H(C', \tilde{q}(C')) = U' + \mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_H) \geq 0,$$

with $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$. Moreover, $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] \geq \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$, where the first inequality follows from $\tau(P')$ being a threshold policy and the second inequality from $\mathcal{Q}^H$ and $\mathcal{Q}^L$ being equivalent measures. Therefore, with $U' \geq 0$ and $k_L < k_H$, we have

$$u_L(\sigma) \geq u_L(C', \tilde{q}(C')) = U' + \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_L) > 0. \quad \square$$

Proof of Proposition 8. By definition, in any separating PBE, $\mathbb{S}_L \cap \mathbb{S}_H = \emptyset$. Hence, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the result follows from Lemma A.3. $\square$

The proof of Proposition 9 requires several additional lemmas and is found in Appendix B.

The High-type Optimal (HTO) Contract To begin, we characterize the high-type optimal equilibrium restricting to pure-strategy, full pooling PBE. We then prove Lemma 5 using a series of auxillary lemmas. Propositions 10-11 follow.

In any pure-strategy, full pooling PBE: $\mathbb{S}_L = \mathbb{S}_H = \{C\}$ for some contract $C = (U, P)$, and $\tilde{q}(C) = q_0$. As discussed in the text, high-type optimality requires that in the contract C we have $U = F_B(q_0|P)$ and P solves (4). We refer to such solutions as *high-type optimal (pooling) contracts*, and use P_H to denote the price component of the solution. As with the proofs for the baseline model, analysis is aided by working with the log-likelihood of belief.

First, we show that if $z < \beta(k_H)$ then $P_H(z) > k_H$ and there is inefficient delay in equilibrium (Lemma A.4), as in the baseline model . This result directly implies that there exists a $\underline{z}^*_H \geq \beta(k_H)$ such that for $z < \underline{z}^*_H$ (too much) delay takes place in equilibrium. Second, we show that there exists a $\bar{z}^*_H$ such that for $z \geq \bar{z}^*_H$ direct completion of the deal is optimal (Lemma A.5). Finally, we prove existence of a lower bound on the high-type seller's reservation value k_H, which if satisfied implies $\bar{z}^*_H = \underline{z}^*_H = \beta(k_H)$ (Lemma A.6).

Lemma A.4. *In a high-type optimal contract, if $q_0 < b(k_H)$ then $P_H(q_0) > k_H$, and hence, execution is delayed. That is, $b(P_H(q_0)) > q_0$.*

Proof. First, define the high-type's payoff for $z < \beta(P)$ as

$$F(z|P) \equiv F_H(z|P) + F_B(z|P)$$
$$= (P - k_H)e^{(u-1)z}\left(\frac{Pu}{(u-1)(V_H - P)}\right)^{1-u} + \frac{Pe^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}}{(u-1)\left(e^z+1\right)}$$

with $u > 1$ and $f(z|P) \equiv \frac{\partial}{\partial P}F(z|P)$. The proof is handled in three steps corresponding to three different regions of the parameter space. Figure 8 illustrates the region of the parameter space for which each different step proves that $f(z|P) > 0$.

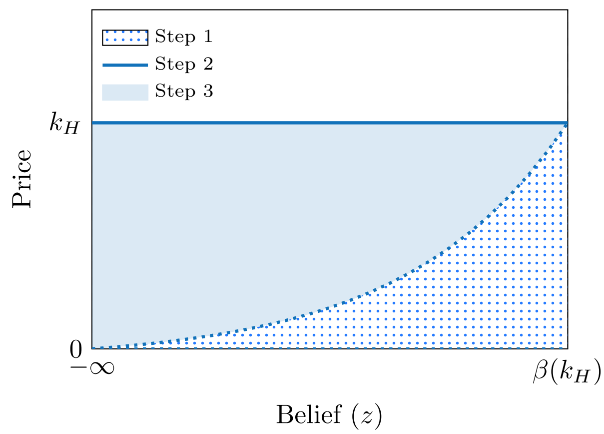


FIGURE 8: Illustration of where in the State Space the Different Steps prove that $f(z|P) > 0$.

Step 1. *For $z < \beta(k_H)$, we have that $f(z|P_0(z)) > 0$.*

Ovserve that F is constant in P for all $P < P_0(z)$. Hence, we can verify the claim by evaluating f at $z = \beta(P)$. Because $u > 1$, direct calculation yields

$$f(\beta(P)|P) = \frac{(u-1)V_H(P - k_H)}{P(P - V_H)} > 0.$$

Step 2. *For $z < \beta(k_H)$, we have that $f(z|k_H) > 0$.*

The proof of the previous step also implies that $f(\beta(k_H)|k_H) = 0$. For $z < \beta(k_H)$, we have that

$$f(z|k_H) = \frac{e^{(u-1)z}\left(\frac{k_H u}{(u-1)(V_H-k_H)}\right)^{-u}\left((u-1)e^z(k_H - V_H) + k_H u\right)}{(u-1)\left(e^z+1\right)\left(V_H - k_H\right)},$$

Notice f has the same sign as $(u-1)e^z(k_H - V_H) + k_H u$, which is decreasing in z. Therefore, $f(z|k_H)$ has exactly one root and is positive for $z < \beta(k_H)$.

Step 3. *For any $z < \beta(k_H)$, $f(z|P)$ has at most one root in P on $P \in (P_0(z), k_H)$.*

Proving this step has several sub-steps:

1. For $z < \beta(P)$ and $P < V_H$, $f(z|P)$ has the same sign as

$$M(z,P) = -\Bigg((u-1)e^z\left(V_H(V_H - k_H u) + P^2 + P(u-2)V_H\right) + u\left(V_H(k_H(-u) + k_H + P(u-2)) + P^2\right)\Bigg). \tag{8}$$

M is quadratic in P, and therefore has at most two roots in P. Furthermore, M is negative as as $P \to \pm\infty$ since the coefficient on P^2 is negative.

2. Evaluating M at $P = k_H$, we get

$$M(z, k_H) = (k_H - V_H)\left(-\left((u-1)e^z(k_H - V_H) + k_H u\right)\right),$$

which is decreasing in z. Furthermore, $M(\beta(k_H), k_H) = 0$. Hence $M(z, k_H)$ is positive for $z < \beta(k_H)$. From 1, $M(z,P)$ is negative for P sufficiently large or small. It follows that $M(z,P)$ has exactly one root for $P < k_H$ and therefore at most one root for $P \in (0, k_H)$.

From Step 1 and Step 2, we know $f(z|P)$ is positive at $P = P_0(z)$ and at $P = k_H$. Step 3 tells us $f(z|P)$ can change signs at most once on the interval $P \in (P_0(z), k_H)$. Combined, we conclude that $f(z|P)$ does not change signs on the interval and therefore is strictly positive for all $P \in (P_0(z), k_H)$, which proves Lemma A.4.

Remark: For $z < b(k_H)$ and $P \in (P_0(z), k_H)$ $f(z|P) = 0$ cannot occur because this would imply $M(z,P)$ is tangent at zero and does not cross it, which is impossible since it is a quadratic equation with negative limits as $P \to \pm\infty$ and $M(z, k_H) > 0$. □

Lemma A.5. *There exists a $\bar{z}_H^*$ such that for $z \geq \bar{z}_H^*$ direct completion is the high-type optimal (pooling) contract.*

Proof. The proof has three steps. First, $F(z|P) \leq \sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)]$.

This follows from

$$\begin{aligned} F(z|P) &= \mathbb{E}_z^H[e^{-r\tau(P)}(P - k_H)] + \mathbb{E}_z[e^{-r\tau(P)}(V(q_{\tau(P)}) - P)] \\ &\leq \mathbb{E}_z^H[e^{-r\tau(P)}(P - k_H)] + \mathbb{E}_z^H[e^{-r\tau(P)}(V(q_{\tau(P)}) - P)] \\ &= \mathbb{E}_z^H[e^{-r\tau(P)}(V(q_\tau) - k_H)] \\ &\leq \sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)], \end{aligned}$$

where the first inequality comes from the fact that the drift of Z is higher under the high-type's probability measure than the acquirer's.

Second, the solution to $\sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)]$ is a threshold policy: stop if and only if $z > z_H^*$. The proof of this result is standard and therefore omitted (see e.g., Dixit et al., 1994, Ch. 5). Third, for $z \geq z_H^*$, the high type optimal contract can achieve the upper bound from the first step with $P = P_0(z)$. □

Lemma A.6. *Assume* $\frac{k_H}{V_H} \geq \left(1 - \frac{u}{2}\right)$*. Then* $\bar{z}_H^* = \underline{z}_H^* = b(k_H)$*.*

Proof. We know that for $z < \beta(P)$ and $P < V_H$, $f(z|P)$ has the same sign as $M(z, P)$, which is quadratic in P (see (8)). Recall from the Proof of Lemma A.4 that $M(z, k_H)$ is decreasing in z and $M(\beta(k_H), k_H) = 0$. Therefore, for $z > \beta(k_H)$, $M(z, k_H)$ is negative.

Observe that

$$\frac{\partial M(z,P)}{\partial P} = -\left((u-1)e^z + u\right)\left(2P + (u-2)V_H\right),$$

which is negative for $P > k_H$ because $2P + (u-2)V_H > 2k_H + (u-2)V_H > 0$. Therefore, for $z > \beta(k_H)$ and $P > P_0(z) > k_H$ we have that $M(z, P) < 0$ and therefore that $f(z|P) < 0$. This result implies that $P > P_0(z)$ is never optimal for $z > \beta(k_H)$.

Lemma A.4 in combination with the previous step completes the proof. □

The High-type Optimal (HTO) Equilibrium

Denote the set of high-type optimal (pooling) contracts as $\mathbb{C}_H(q_0)$ and the high type's PBE payoff from pooling on a solution as $u_H^*(q_0)$. If a contract will result in immediate execution, then the distinction between U and P is irrelevant as only $U + P$ matters. In this case, we simplify exposition by setting $P = P_0(q_0)$. If there exist contracts that induce immediate execution given q_0, define $C_i(q_0) \equiv (P_0(q_0), \mathbb{E}_{q_0}[V_\theta] - P_0(q_0))$ as the high-type optimal contract that results in immediate execution. If it exists, define $C_d(q_0) \equiv \arg\max_{\{C: P > P_0(q_0)\}} U + F_H(q_0|P)$, which is the high-type optimal contract that results in delay given q_0.

Lemma A.7. *The set of high-type optimal contracts has the following properties:*

1. *If $C_d(q_0)$ exists, then it is unique.*
2. *$\mathbb{C}_H(q_0)$ is one of the following:*
   (a) *$\mathbb{C}_H(q_0) = \{C_i(q_0)\}$, i.e., direct execution is uniquely optimal.*
   (b) *$\mathbb{C}_H(q_0) = \{C_d(q_0)\}$, i.e, delay is uniquely optimal*
   (c) *$\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$.*
3. *If $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$, then $u_L(C_i(q_0), q_0)) > u_L(C_d(q_0), q_0))$.*
4. *For any $C \in \mathbb{C}_H(q_0)$, $u_\theta(C, q) > 0$ is nondecreasing in q.*

5. *The problem's value $u_H^*(q_0)$ is increasing in q_0.*

Proof. Taking each in turn:

1. From the proof of Lemma A.4 it follows that the first-order derivative of the seller's payoff (assuming there is delay $q < b(P)$) has the same sign as a function that is a second-order polynomial in P. This implies that for $P > P_0(q_0)$ there can be at most one local maximum for $P \in (P_0(q_0), V_H)$. The fact that there is one local maximum for $P > P_0(q_0)$ proves the result

2. This result follows directly from the previous result.

3. We know that

$$\begin{aligned} u_L(C_i(q_0), q_0) + (k_L - k_H) &= u_H(C_i(q_0), q_0) \\ &= u_H(C_d(q_0), q_0) \\ &> u_L(C_d(q_0), q_0) + \mathbb{E}_{q_0}^L \left[e^{-r\tau(P)}(k_L - k_H) \right]. \end{aligned}$$

   The first equality follows from the fact that payoffs only differ because of the different reservation values. The second equality follows from the fact that the high-type seller is indifferent, and the third equality follows from the fact that $P \geq k_H > k_L$ and therefore worse dynamics of beliefs make the seller worse off (the upfront is fixed). This result implies that

$$u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0) + \mathbb{E}_{q_0}^L \left[e^{-r\tau(P)}(k_L - k_H) \right] - (k_L - k_H) > u_L(C_d(q_0), q_0).$$

   The last inequality follows from the fact that $\tau(P) > 0$ and $k_H > k_L$ and therefore

$$\mathbb{E}_{q_0}^L \left[e^{-r\tau(P)}(k_L - k_H) \right] > (k_L - k_H).$$

4. Because $C \in \mathbb{C}_H(q_0)$, we know that $P \geq k_H > k_L$ and therefore the seller's payoff upon completion is non-negative. The upfront is fixed. Therefore, higher beliefs q weakly improve $u_\theta(C, q)$.

   The fact that $P \geq k_H > k_L$ implies that the seller's payoff (without the upfront) is non-negative. Furthermore, the upfront is always positive since $P < V_H$ and therefore $q_\theta(C, q) > 0$.

5. Take any $C \in \mathbb{C}_H(q_0)$. Fixing the price $P < V_H$, the upfront is increasing in beliefs. Furthermore, the seller's expected payoff upon execution is non-decreasing in beliefs since $P \geq k_H > k_L$ therefore the seller must be better of with higher beliefs. □

Definition 1. *Let $\mathbb{H}(q_0)$ be the set of pure-strategy, full pooling PBE in which both types pool on some contract $C \in \mathbb{C}_H(q_0)$.*

Lemma A.8. $\mathbb{H}(q_0)$ *is the set of HTO equilibria.*

Proof. It is straightforward to verify that any $\sigma^* \in \mathbb{H}(q_0)$ is a PBE when paired with off-path beliefs $\tilde{q}(C') = 0$ for all deviations C'. Turning to the payoffs, by definition, $u_H(\sigma^*) = u_H^*(q_0) > 0$ for any $\sigma^* \in \mathbb{H}(q_0)$. Consider now PBE $\sigma \notin \mathbb{H}(q_0)$. Let $\underline{q}_\theta = \min_{\mathbb{S}_\theta} \tilde{q}(C)$. If $\underline{q}_H < q_0$, then there exists $C \in \mathbb{S}_H$ such that,

$$u_H(\sigma) = u_H(C, \underline{q}_H) \leq u_H^*(\underline{q}_H) < u_H^*(q_0),$$

where the last inequality is from Lemma A.7(5). If $\underline{q}_H > q_0$, then Bayesian consistency requires $\underline{q}_L = 0$. Therefore, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the equilibrium is trivial by Lemma A.3, so $u_H(\sigma) = 0 < u_H^*(q_0)$. This establishes that $u_H(\sigma) < u_H^*(q_0)$ for any PBE σ in which $\underline{q}_H \neq q_0$.

So, if $u_H(\sigma) \geq u_H^*(q_0)$, then $\underline{q}_H = q_0$. Bayesian consistency then requires $\tilde{q}(C) = q_0$ for all $C \in \mathbb{S}_H \cup \mathbb{S}_L$, meaning both types play the same strategy. By definition, to achieve $u_H^*(q_0)$ their common support must be a subset of $\mathbb{C}_H(q_0)$. If $\mathbb{C}_H(q_0)$ is a singleton, this concludes the proof. Otherwise, by Lemma A.7(2), $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$. In this case, the seller cannot mix between the two solutions, because $u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0)$ by Lemma A.7(3). □

Proof of Lemma 5. Follows from Lemma A.8. □

Proof of Proposition 10. Follows from Lemmas A.4-A.6 and A.8. □

Proof of Proposition 11. Follows from Lemmas A.4, A.5, and A.8. □

A.4 Proofs for Section 5

Definition of Equilibrium with Dynamic Bidding: A quadruple $\{P, \nu, \tau, q\}$ such that

1. *Seller optimality:* $\nu(t) \in \arg\sup_{T \geq t} \mathbb{E}_t[e^{-r(T+\tau(P_T,T)-t)}(P_T - k)]$. Let $G_t(\omega)$ denote the seller's payoff under the solution.

2. *Acquirer optimality:* given any offer P accepted at any date t, $\tau(P,t)$ solves the acquirer problem $\sup_{\tau \geq t} \mathbb{E}_t[e^{-r(\tau-t)}(V(q_\tau) - P)]$

3. *Buyer competition:* Given the seller's continuation payoff at any $t \leq \nu$, $G_t(\omega)$, there does not exist an offer y such that $G_t(\omega) < \mathbb{E}_t[e^{-r(\tau(y,t)-t)}(y - k)]$.

Proof of Proposition 12. Recall that the seller's payoff in the baseline model is the same as in the solution to (HSP), which we denote by $F_0(q)$. Therefore, it suffices to show that the seller's payoff in any equilibrium with dynamic bidding is equal to $F_0(q)$. Consider the No Deals condition and let $y = P_S(q)$, then immediately we have $G_t(\omega) \geq \mathbb{E}_t[e^{-r(\tau(P_S(q_t),t)-t)}(P_S(q_t) - k)] = F_S(q_t|P_S(q_t)] = F_0(q_t)$. Setting $t = 0$, we conclude that the seller's payoff with dynamic bidding is weakly bigger than with static bidding.

Next, we argue $G_t(\omega) \le F_0(q_t)$. Fix any $\nu \ge t$. Using the tower property, we have

$$\mathbb{E}_t[e^{-r(\nu-t)}e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] = \mathbb{E}_t[e^{-r(\nu-t)}\mathbb{E}_\nu[e^{-r\tau(P_\nu,\nu)}(P_\nu - k)]]$$

And clearly $\mathbb{E}_\nu[e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] \le \max_P\{\mathbb{E}_\nu[e^{-r\tau(P,\nu)}(P - k)]$, therefore

$$\begin{aligned}\mathbb{E}_t[e^{-r(\nu-t)}e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] &\le \mathbb{E}_t[e^{-r(\nu-t)}\max_P\{\mathbb{E}_\nu[e^{-r\tau(P,\nu)}(P - k)]\}] \\ &= \mathbb{E}_t[e^{-r(\nu-t)}\mathbb{E}_\nu[F_S(q_\nu|P_S(q_\nu))]] \\ &= \mathbb{E}_t[e^{-r(\nu-t)}F_0(q_\nu)] \\ &\le F_0(q_t).\end{aligned}$$

where the first equality is by the definition of P_S, the second equality follows from the previously established fact that $F_S(q|P_S(q)) = F_0(q)$. The last inequality follows because ν acts as a constraint and any constraint on the stopping rule (i.e., $\tau \ge \nu$) in (HSP) is dominated by the solution to the unconstrained problem. Taking the supremum over all ν, we conclude that $G_t(\omega) \le F_0(q_t)$.

We have therefore established that for any t, $F_S(q_t|P_S(q_t)] \le G_t(\omega) \le F_0(q_t) = F_S(q_t|P_S(q_t)]$. Letting $t = 0$ completes the proof. □

Lemma A.9. *Suppose that $k < V_L < V_H$. Then, there exists a $\underline{\gamma} > 0$ such that if and only if $\phi^2/r = \gamma \le \underline{\gamma}$, then for any z_0 the winning offer is $P_0(z_0)$ and the transaction is executed immediately.*

Proof. Observe that the seller's payoff for $z \le b(P)$ can be written as

$$F_S(z|P) = \psi(P,u)\frac{e^{uz}}{1+e^z}$$

with

$$\psi(P,u) = (P-k)\left(\frac{u(V_L - P)}{(u-1)(P-V_H)}\right)^{-u}\left(\frac{u(V_L-P)}{(u-1)(P-V_H)} + 1\right).$$

Given any z, picking the optimal price boils down to solving

$$\max_P F_S(z|P) \iff \max_{P \ge P_0(z)} \psi(P,u) \tag{9}$$

since the seller either picks the highest price that leads to direct execution $P_0(z)$ or a higher price that leads to delay.

For any belief it is optimal to have direct execution (pick $P_0(z)$) if and only if

$$\psi_P(P,u) \le 0.$$

Assume that $\psi_P(P,u) \leq 0$ then the optimal solution to equation (9) is $P_0(z)$. Assume that direct execution is optimal for all z but $\psi_P(P,u) > 0$ for some $\hat{P}$. Take $\hat{z}$ such that $\hat{P} = P_0(\hat{z})$ then for a small $\epsilon > 0$

$$\psi(\hat{P}+\epsilon, u) > \psi(\hat{P}, u)$$

since $\psi_P(P,u)$ is continuous in P and $\psi_P(\hat{P},u) > 0$. This result contradicts the fact that direct execution is optimal for $\hat{z}$. Therefore, if direct execution is optimal $\psi_P(P,u) \leq 0$.

Observe that $\psi_P(P,u)$ for $P \in (V_L, V_H)$ has the same sign as

$$\begin{aligned}\tilde{\psi}_P(P,u) =& \psi_P(P,u)(u-1)(P-V_H)^2(P-V_L)\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^u \\ =& k(u-1)u(V_H-V_L)^2 - P^3 + P^2(-uV_H + uV_L + 2V_H + V_L) \\ & - P\left(2\left(-u^2+u+1\right)V_HV_L + u^2V_L^2 + (u-1)^2V_H^2\right) + V_HV_L(-uV_H + uV_L + V_H).\end{aligned}$$

Further, observe that for $P \in (V_L, V_H)$

$$\begin{aligned}\tilde{\psi}_{P,u,P}(P,u) &= -2(V_H-V_L)(P+(u-1)V_H - uV_L) < 0, \\ \tilde{\psi}_{P,u}(V_L,u) &= (2u-1)(k-V_L)(V_H-V_L)^2 < 0,\end{aligned}$$

which establishes that $\tilde{\psi}_{P,u}(P,u)$ is negative at $P = V_L$ and decreasing in P. Hence, it is negative for all for $P \in (V_L, V_H)$. It also follows from above that if $\tilde{\psi}_P(P,u) \leq 0$ for $P \in (V_L, V_H)$ then $\tilde{\psi}_P(P,u') \leq 0$ for any $u' \geq u$ and $P \in (V_L, V_H)$.

The relationship between $\psi_P(P,u)$ and $\tilde{\psi}_P(P,u)$ then implies that if for some $\hat{u}$ $\psi_P(P,\hat{u}) \leq 0$ for $P \in (V_L, V_H)$ then for any $u \geq \hat{u}$ $\psi_P(P,u) \leq 0$ for $P \in (V_L, V_H)$.

Clearly, u is decreases with γ. Therefore, if for some $\hat{\gamma}$ $\psi_P(P, u(\hat{\gamma})) \leq 0$ for $P \in (V_L, V_H)$ then for any $\gamma \leq \hat{\gamma}$ $\psi_P(P, u(\gamma)) \leq 0$ for $P \in (V_L, V_H)$. The previous steps show that direct execution is optimal for any belief if and only if $\gamma \leq \underline{\gamma}$.

We can collect the terms related to u in $\tilde{\psi}_P(P,u)$ and obtain

$$\begin{aligned}\tilde{\psi}_P(P,u) =& -u(V_H-V_L)\left(k(V_H-V_L) + P^2 - 2PV_H + V_HV_L\right) \\ & + u^2(k-P)(V_H-V_L)^2 - P^3 + 2P^2V_H + P^2V_L - PV_H^2 - 2PV_HV_L + V_H^2V_L.\end{aligned}$$

As $\gamma \to 0$, $u \to \infty$ and

$$\tilde{\psi}_P(P,u) \to u^2(k-P)(V_H-V_L)^2 < 0.$$

Therefore, there exists a γ small enough such that $\tilde{\psi}_P(P,u) < 0$ and direct execution is optimal. □

Lemma A.10. *Suppose that $k < V_L < V_H$ and $\phi^2/r > \underline{\gamma}$ then there exist two thresholds $z_a < z_b$ such that*

(a) For $z_0 \in (z_a, z_b)$, the winning offer is $P_0(z_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(z_b))$.

(b) For $z_0 \notin (z_a, z_b)$, the winning offer is $P_0(z_0)$ and the transaction is executed immediately.

Proof. Assume $\gamma > \underline{\gamma}$ then we know delay takes places for some z, see Lemma A.9. Assume delay takes place for $\hat{z}$ and the seller picks a price $\tilde{P} > P_0(\hat{z})$ then define $\tilde{z}$ as the solution to $\tilde{P} = P_0(\tilde{z})$. We then know that delay must take place for $z \in (\hat{z}, \tilde{z})$ since

$$\psi(\tilde{P}, u) = \max_{P \geq P_0(\hat{z})} \psi(P, u) \geq \max_{P \geq P_0(z)} \psi(P, u).$$

Assume there is more than one region in which delay takes place. Assume for simplicity there are two. The proof also works with more regions. The existence of these two delay regions implies that there exists a $z_1 < z_2 < z_3 < z_4 < z_5$ such that

1. For $z < z_1$, direct execution is optimal because $\lim_{P \to V_L} \psi(P, u) = \infty$ and $\psi(P, u) < \infty$ for any $P \in (V_L, V_H)$ and therefore for $z < z_1$ $\psi_P(P_0(z), u) \leq 0$.

2. For (z_1, z_2), delay takes place and therefore there exists a $z \in (z_1, z_2)$ such that $\psi_P(P_0(z), u) > 0$.

3. For (z_2, z_3), no delay takes place and therefore for $z \in (z_2, z_3)$ $\psi_P(P_0(z), u) \leq 0$.

4. For (z_3, z_4), delay takes place and therefore there exists a $z \in (z_3, z_4)$ such that $\psi_P(P_0(z), u) > 0$.

5. For $z > z_5$, no delay takes since $\lim_{P \to V_H} \psi_P(P, u) = (u-1)u(k - V_H)(V_H - V_L)^2 < 0$

This implies that $\psi_P(P, u) = 0$ at least 5 values of $P \in (V_L, V_H)$.

We know that $\hat{\psi}_P(P, u)$ has the same sign as $\psi_P(P, u)$. Observe that $\hat{\psi}_P(P, u)$ is a third-degree polynomial in P, which has at most three zero solutions and therefore it must be the case that we only have one region in which delay takes place. □

Proof of Proposition 13. The result follows directly from Lemma A.9 and Lemma A.10. □

Proof of Proposition 14. For $\gamma > \underline{\gamma}$, delay is optimal and dynamic bidding strictly dominates static bidding for some beliefs. Therefore, there must exist a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$ dynamic bidding strictly dominates static bidding. □

Proof of Lemma 6. As before, the smooth pasting condition determines the optimal exercise threshold b_d given d

$$\beta_d(F) = \ln\left(\frac{u}{u-1}\right) + \ln\left(\frac{A}{V_H - F}\right).$$

Invert $b_d(F)$ (i.e., $\beta_d(F)$ converted to a probability) to obtain the direct execution price for any given belief

$$P_D(q) = qb_d^{-1}(q) + (1-q)A = -\frac{A(1-q)}{u-1} + qV_H$$

Then

$$P_D(q) - P_0(q) = (1-q)\left(-\frac{A}{u-1} + \frac{qV_H}{u-q}\right),$$

which is strictly positive for $q \geq k/V_H$ and $A < \bar{A} = k(u-1)/(u-k/V_H)$. □

B Internet Appendix

B.1 Additional Proofs for Section 4

Proof of Proportion 9. Lemma A.8 establishes that $\mathbb{H}(q_0)$ is the set of HTO equilibria. To establish Proportion 9, we use the following specification of off-path beliefs. Define $B_\theta(C, u) \equiv \{q : u_\theta(C, q) \geq u\}$. For any $\sigma \in \mathbb{H}(q_0)$ with $C^* \in \mathbb{C}_H(q_0)$ as the pooling contract, specify the off-path beliefs as follows. For any $C' \neq C^*$: $\tilde{q}(C') = q_0$ if $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, and $\tilde{q}(C') = 0$ otherwise, where $\subset$ denotes strict inclusion. To verify σ remains a PBE, note that neither type wishes to deviate to C' with $\tilde{q}(C') = 0$. In addition, the high type has no incentive to deviate to C' with $\tilde{q}(C') = q_0$ as $u_H(C', \tilde{q}(C')) = u_H(C', q_0) \leq u_H^*(q_0) = u_H(\sigma)$. By construction, at such C', $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, so the low type has no incentive to deviate either. Moreover, since $u_H(\sigma) = u_H^*(q_0) > 0$ (by Lemma A.7(4)) the equilibrium is nontrivial, and the beliefs are constructed to satisfy divinity.

The remainder of the proof is handled by Lemmas B.1-B.3 below. □

Lemma B.1. *If $\sigma \in \mathbb{H}(q_0)$, then σ is undefeated.*

Proof. Let $\sigma \in \mathbb{H}(q_0)$ with off-path beliefs as specified at the start of the proof of the proposition. For the purpose of contradiction, suppose there exists PBE σ' that defeats σ. By Lemma A.8, $u_H(\sigma) \geq u_H(\sigma')$. The following are then required for σ' to defeat σ: there exist $C' \in \mathbb{S}'_L$ where $C' \neq C^*$, and $u_L(\sigma') > u_L(\sigma) \geq 0$. Hence, by Lemma A.3, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C' \in \mathbb{S}'_H$ as well. For σ' to defeat σ then further requires that $u_H(\sigma') \geq u_H(\sigma)$. So, we have $u_H(\sigma') = u_H(\sigma)$. In this case, σ' defeating σ requires that $\tilde{q}(C') \notin [0, q_0]$. However, as specified above, $\tilde{q}(C) \in [0, q_0]$ for all C, which is a contradiction. □

Lemma B.2. *If there exist PBEs $\sigma \in \mathbb{H}(q_0)$ and $\sigma' \notin \mathbb{H}(q_0)$, such that $u_L(\sigma') > u_L(\sigma)$, then σ' does not satisfy Divinity.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* \in \mathbb{C}_H(q_0)$. From Lemma A.7(4), $u_\theta(C^*, q)$ is nondecreasing in q for both θ. Hence, for both types, $B_\theta(C^*, u_\theta(\sigma'))$ is an interval $[\underline{b}_\theta, 1]$, where $\underline{b}_\theta$ denotes the lowest q-value such that $u_H(C^*, \underline{b}_\theta) \geq u_\theta(\sigma')$. In addition, $\underline{b}_H \leq q_0 < \underline{b}_L$, where the first inequality is from the hypothesis that $\sigma' \notin \mathbb{H}(q_0)$ and the second inequality is from the hypothesis that $u_L(\sigma') > u_L(\sigma)$. If C^* is off-path under σ', Divinity then requires that $\tilde{q}(C^*) \geq q_0$ and the high type would profit by deviating to C^*, breaking the PBE. Hence, it is sufficient to establish that C^* is off-path under σ'.

Suppose C^* is on-path in σ'. By $u_L(\sigma') > u_L(\sigma) > 0$ and Lemma A.3, σ' is nontrivial, so $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and $u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$. From Lemma A.7(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma A.3), which is a contradiction. □

Lemma B.3. *If there exist nontrivial PBEs $\sigma \in \mathbb{H}(q_0)$ and $\sigma' \notin \mathbb{H}(q_0)$, such that $u_L(\sigma') \leq u_L(\sigma)$, then σ that defeats σ'.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* = (U^*, P^*) \in \mathbb{C}_H(q_0)$. By definition, σ defeats σ' if: (i) C^* is off-path in σ'; (ii) $u_\theta(\sigma) \geq u_\theta(\sigma')$ for both θ, and holding strictly for at least one θ; and (iii) $\tilde{q}'(C^*) \notin [q_0, 1]$. Requirement (ii) holds for $\theta = H$ by Lemma A.8 and for $\theta = L$ by hypothesis. Moreover, if requirement (iii) fails, then because $u_H(C^*, q)$ is nondecreasing in q (Lemma A.7(4)),

$$u_H(\sigma') \geq u_H(C^*, \tilde{q}'(C^*)) \geq u_H(C^*, q_0) = u_H(\sigma),$$

which contradicts Lemma A.8. Hence, it is sufficient to show that (i): C^* is off-path in σ'.

Suppose C^* is on-path in σ'. Since σ' is nontrivial, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and $u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$, where the inequality is from Lemma A.8. From Lemma A.7(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma A.3), which is a contradiction. □

B.2 When Due Diligence Information is Contractible

In this subsection, we analyze and prove the main result for the model in which due diligence information is contractible. The space of contracts are now pairs $\mathcal{C} = \{U, (P_t)_{t\geq 0}\}$ where U is the upfront transfer and P_t is the execution-contingent price, which can depend on the entire history of information, $\mathcal{H}_t = \sigma(\{X_s : 0 \leq s \leq t\})$ (i.e., P_t is measurable with respect to $\mathcal{H}_t$).

Consider an arbitrary contract $\mathcal{C}$ and execution rule τ. Abusing notation, let $F_\theta(\mathcal{C}, \tau) = \mathbb{E}_q^\theta[e^{-r\tau}(P_\tau - k_\theta)]$ denote the type-θ seller's payoff in the due diligence subgame, and let $F_B^\theta(\mathcal{C}, \tau) = \mathbb{E}_q^\theta[e^{-r\tau}(V_\theta - P_\tau)]$ denote the acquirer's payoff in the due diligence subgame conditional on the seller's type being θ.

The high-type optimal separating contract solves

$$\sup_{\mathcal{C},\tau} F_H(\mathcal{C}, \tau) + U$$

$$s.t. \quad F_L(\mathcal{C}, \tau) + U \leq 0 \tag{10}$$

$$F_B^H(\mathcal{C}, \tau) - U \geq 0 \tag{11}$$

$$\tau \in \arg\sup_T \mathbb{E}_q^H[e^{-rT}(V_H - P_T)]. \tag{12}$$

The first constraint, (10), says that the low type prefers to reject the contract.[29] The

[29]We will see that it is unnecessary to provide any surplus to the low-type seller to achieve separation.

second constraint, (11), says that the acquirer is willing to enter the contract knowing the asset is high value. The third constraint, (12), ensures that τ is consistent with acquirer optimization in the due diligence subgame.

It is easy to see that both (10) and (11) must bind in any solution. Thus, $F_B^H = U$ and the problem can be written as

$$\sup_{\mathcal{C},\tau} F_H(\mathcal{C},\tau) + F_B^H(\mathcal{C},\tau)$$

$$s.t. \quad F_L(\mathcal{C},\tau) = -F_B^H(\mathcal{C},\tau) \tag{13}$$

$$\tau \in \arg\sup_T \mathbb{E}_q^H[e^{-rT}(V_H - P_T)]. \tag{14}$$

Note that the objective is equal to the total surplus, $F_H(\mathcal{C},\tau)+F_B^H(\mathcal{C},\tau) = \mathbb{E}^H\left[e^{-r\tau}(V_H - k_H)\right].$ So, the problem boils down to finding the stopping time that maximizes $\mathbb{E}^H[e^{-r\tau}]$ (which is bounded above by 1), subject to finding prices that satisfy the two constraints.

Approximate Solution Given that the high type prefers to accept the contract, if the acquirer immediately executed the transaction, then the low type would also prefer to accept. Thus, to dissuade the acquirer from immediate execution, we set the price above V_H until the first time that X_t exits an ε-neighborhood around the initial starting point. Denote this time by $\tau(\varepsilon) \equiv \inf\{t : |X_t - X_0| \geq \varepsilon\}$.

Proposition B.1. *For any $\varepsilon > 0$, there exists a $\mathcal{C}_\varepsilon$ such that $(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ satisfies (13) and (14) in which the high-type seller extracts all of the surplus. Moreover,* $\lim_{\varepsilon\to 0} \mathbb{E}^H[e^{-r\tau(\varepsilon)}] = 1.$

Proof of Proposition B.1. We prove the first statement by construction. Define P_t^ε as follows:

$$P_t^\varepsilon = \begin{cases} \underline{P}^\varepsilon & t \geq \tau(\varepsilon) \text{ and } X_{\tau(\varepsilon)} = x_0 - \varepsilon \\ V_H & \text{otherwise} \end{cases}$$

Let $U^\varepsilon = \mathbb{E}^H[e^{-r\tau(\varepsilon)}(V_H - P_{\tau(\varepsilon)})]$. We claim that there exists a $\underline{P}^\varepsilon$ such that $(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ satisfies (13) and (14). Satisfying (14) is trivial: any $\underline{P}^\varepsilon \leq V_H$ suffices.[30] For (13), constructing the payoffs $F_B^H(\mathcal{C}_\varepsilon, \tau(\varepsilon))$ and $F_L(\mathcal{C}_\varepsilon, \tau(\varepsilon))$, it is straightforward to verify that both F_L and $-F_B^H$ are linearly increasing in $\underline{P}^\varepsilon$ with respective slopes $\frac{e^{(q_1+q_2)\varepsilon}}{e^{q_1\varepsilon}+e^{q_2\varepsilon}} > \frac{e^{(u_1+u_2)\varepsilon}}{e^{u_1\varepsilon}+e^{u_2\varepsilon}}$. Hence, there exists a unique P_0^ε satisfying (13). To see that this $P_0^\varepsilon \leq V_H$, and thus is consistent with (14),

Hence, it is sufficient to consider contracts in which the low types earns at most zero from accepting the contract.

[30] For any $\underline{P}^\varepsilon < V_H$, it will be strictly optimal to execute at $\tau(\varepsilon)$ if $q_{\tau(\varepsilon)} = q_0 - \varepsilon$ since the price is fixed thereafter and delay is costly. The acquirer strictly prefers not to execute before $\tau(\varepsilon)$, since the payoff from doing so is zero and there is positive probability of a positive payoff from not doing so. And the acquirer is willing to execute at $\tau(e)$ if $q_{\tau(\varepsilon)} = q_0 + \varepsilon$ since her payoff from any stopping rule thereafter is zero.

observe that at $\underline{P}^{\varepsilon} = V_H$: F_L is strictly positive whereas $-F_B^H = 0$, which therefore implies the intersection (i.e., P_0^{ε}) must lie below V_H.

For the second statement (i.e., that $\lim_{\varepsilon \to 0} \mathbb{E}^H[e^{-r\tau(\varepsilon)}] = 1$) given any initial $X_0 = x$, let $g(x) = E_x^H[e^{-r\tau(\varepsilon)}]$. Then $rg(x) = g'(x) + \frac{1}{\phi^2} g''(x)$ and $g(x+\varepsilon) = g(x-\varepsilon) = 1$. Solving gives

$$g(x) = \frac{\left(e^{\varepsilon\phi^2} + 1\right) e^{\frac{1}{2}\varepsilon\phi^2\left(\sqrt{\frac{4r}{\phi^2}+1}-1\right)}}{e^{\varepsilon\phi^2\sqrt{\frac{4r}{\phi^2}+1}} + 1}.$$

Observe that both the enumerator and denominator converge to 1 as $\varepsilon \to 0$. □