Due Diligence

BRENDAN DALEY, THOMAS GEELEN, AND BRETT GREEN*

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ABSTRACT

Due diligence is common practice prior to the execution of large transactions. We propose a model of due diligence and analyze its effect on prices, payoffs, and deal completion. In our model, if the seller accepts an offer, the acquirer has the right to gather information and chooses when to execute the transaction. In equilibrium, the acquirer engages in "too much" due diligence. Our quantitative results suggest the magnitude of the distortion is economically significant. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting without due diligence. We use our framework to explore the timing of due diligence, bidder heterogeneity, and break-up fees.

Keywords: Due Diligence, Learning, Takeovers, Mergers and Acquisitions.

JEL Classification: C7, D4, G0.

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*Brendan Daley is at Johns Hopkins University. Thomas Geelen is at Copenhagen Business School. Brett Green (corresponding author, b.green@wustl.edu) is at Washington University in St. Louis. Geelen gratefully acknowledges support from the Danish Finance Institute and the Center for Financial Frictions (FRIC), grant no. DNRF102. The authors are grateful to Philip Bond, an anonymous associate editor, and two anonymous referees for their valuable feedback and suggestions. We are grateful to Bruno Biais, Will Cong, Jason Donaldson, Alexander Gorbenko, Stefano Lovo, Andrey Malenko, Simon Mayer, Lasse Heje Pedersen, and Giorgia Piacentino for valuable comments. We also thank seminar participants at the AFA 2022, the Cambridge Corporate Finance Theory Conference 2021, Columbia University, Copenhagen Business School, Cornell University, Duke University, HEC Paris, Princeton University, the SFS Cavalcade 2021, University of Toronto, University of Utah, Vienna University of Economics and Business, Washington University in St. Louis, the FTWebinar, and the Virtual Finance Theory Seminar. We are indebted to Cyrus Mevorach for outstanding research assistance. We have read the Journal of Finance disclosure policy and have no conflicts of interest to disclose.

Due diligence is pervasive. It is perhaps most salient within the context of corporate acquisitions and real estate transactions. However, a period of "discovery" prior to the transfer of ownership and during which the acquirer can gather information extends well beyond these two realms. Practitioners argue that due diligence is critical to ensure a successful transaction.[1] Third-parties (e.g., a financier) often require some due diligence prior to deal completion. But is it economically important? What are the welfare implications? More specifically, how does the acquirer's ability to conduct due diligence prior to executing a transaction affect deal terms, the likelihood of deal completion, the total surplus and how it is divided? In this paper, we propose and analyze a model of the due diligence process to answer these questions.

The due diligence process is an underexplored area in the literature relative to its prominence in practice. This can likely be attributed to a lack of structured data. Empirical work on Mergers and Acquisitions (M&A) often relies on hand-collected data from SEC filings (Boone and Mulherin, 2007; Liu and Officer, 2019).[2] Progress in textual analysis and the availability of new datasets covering private transactions (e.g., PitchBook, Dealogic, and Preqin) suggest the possibility for more empirical work in this area. One of our aims is to provide a theoretical framework for such empirical work.

Due diligence is inherently a dynamic problem. Both the amount and the type of information collected during the process will depend on what the acquirer has learned to date. If a potentially troubling matter is uncovered in the early stages of due diligence (e.g., a pending lawsuit), then the acquirer will spend additional resources investigating the matter or may cancel the deal entirely. If no such issues arise, the transaction will be executed sooner. For practitioners, Snow (2011) describes "the goal (of due diligence) is to make the buyer comfortable enough to go through with the deal and close."

We therefore model due diligence as a stopping problem during which the acquirer

[1] See e.g., Snow (2011), Lajoux (2010), and Forbes article dated March 27, 2019: "A comprehensive guide to due diligence issues in mergers and acquisitions" (date accessed: August 24, 2020).

[2] More specifically, the "Background to the Offer" section of DEFM14A and Sch 14D1 proxy filings contains rich details about the deal completion process, but it is text-heavy and is not organized in a pre-defined manner.

uncovers information about the asset being acquired. In our baseline model, there is symmetric imperfect information about bidders' (common) value for the seller's asset and thus whether there are gains from trade. Prior to due diligence, bidders make competing price offers for the asset. If the seller accepts one of the offers then the winning bidder (the *acquirer*) has the right to conduct due diligence and decide when (if ever) to execute the transaction at the bid price. Conducting due diligence takes time, and delay costs are captured by time discounting. Thus, the acquirer faces a real option problem during due diligence where the strike price is determined by the winning bid.

The solution to the acquirer's problem is to execute the transaction when the expected value of the asset is above a threshold. This *execution threshold* is increasing in the price: the higher is the price, the more due diligence the acquirer conducts. A higher execution threshold implies a longer due diligence period and a higher probability of deal failure. As a result, the seller's payoff is not monotonically increasing in the price. Rather, it is initially increasing but eventually decreasing, thereby illustrating the trade-off between a higher payoff conditional on execution versus a longer and less promising due diligence process.

In the unique equilibrium, bidders offer the seller's preferred (interior) price and make a positive expected profit despite being perfectly competitive. Due diligence occurs if and only if the prior belief about the value of the asset is below equilibrium execution threshold. Above the threshold, the transaction is executed immediately at the highest price such that the acquirer is willing to forgo due diligence. This price is below the acquirer's expected value for the asset, meaning the option to conduct due diligence confers surplus to the acquirer even when it is not exercised.

We perform comparative statics on the speed with which the acquirer learns during due diligence. For priors below the execution threshold, both the equilibrium price and the execution threshold increase with learning speed, while the likelihood of deal completion falls. Conditional on deal completion, the expected time to completion decreases with learning speed when the prior is low, but increases with learning speed for higher priors.

We compare the equilibrium outcome to two benchmarks: socially optimal due diligence and no due diligence. Because the equilibrium price is above the seller's reservation value, the acquirer's execution threshold is above the socially optimal one. As a consequence, there is "too much" due diligence and too many deals fail. Relative to the model without due diligence, due diligence increases surplus when the prior is low, but decreases social surplus for intermediate priors (i.e., near the socially efficient threshold).

In Section II, we explore the timing of due diligence by allowing due diligence to begin prior to an offer being accepted. We provide a sufficient condition under which the timing of due diligence is irrelevant. This result emphasizes that the crucial assumption is that the acquirer has the option to conduct due diligence after an offer is accepted, not that this option is necessarily exercised. We then analyze the model with common knowledge of gains from trade (e.g., due to synergies). In this case, the aforementioned sufficient condition is violated and the seller strictly prefers that due diligence be performed before accepting an offer. We map these two cases to different types of acquisitions and discuss the empirical implications.

In Section III, we extend the baseline model to allow for bidder heterogeneity. More specifically, we introduce a *strong* bidder who has a higher valuation for the asset than the other (*weak*) bidder. In addition to generalizing our results from the previous section, we demonstrate three new results. First, in equilibrium, the seller always accepts the strong bidder's offered price, despite the fact that it is (strictly) less than the weak bidder's offer (when the prior is below the weak bidder's execution threshold). In essence, the strong bidder is able to extract surplus from the seller via a lower price through the implicit promise of more expedient due diligence. Second, the strong bidder's bid is decreasing in her valuation for the asset: the stronger she is, the larger are the price concessions she can extract from the seller. Finally, total surplus decreases as the weak bidder becomes more competitive with the strong bidder, which contrasts with the usual intuition that more competition leads to a more efficient outcome.

In Section IV, we use our framework to discuss other contractual features and considerations including unconditional transfers, break-up fees, flow costs of conducting due diligence, and the risk of deal failure for exogenous reasons.

Section V then explores the quantitative implications of our model within the context of public M&A. Using empirical moments from the M&A literature, we estimate the parameters of our baseline model. The baseline model does a good job matching the empirical moments. However, it requires an unreasonably high discount rate to do so, which suggests that the discount rate is proxying for other costs associated with delay. We therefore also estimate the two variations of the model with additional sources of costly delay from Section IV: flow costs and exogenous failure rates. Both of these variations of the model can match the data well at a 10% discount rate. Using the estimated model parameters, we conduct two counterfactual exercises. Compared to the social optimum, the economic loss from inefficient due diligence is non-trivial: it corresponds to 4-8% of the total deal value. Moreover, the scope for contractual features (i.e., unconditional transfers and break-up fees) to mitigate the distortion is limited.

A. Due Diligence In Practice

A key assumption of our model is that the acquirer has the right to conduct due diligence after agreeing to terms with the seller, which comports with what we observe in practice. There are good economic reasons for this timing when considering the information acquisition and provision costs, though we will largely abstract from such costs in our analysis.

According to Lajoux (2010), "Some buyers spend millions of dollars identifying every possible risk before signing on the dotted line." For mergers in the financial sector, Cole et al. (2016) document that expenditures on accounting and legal firms alone average 47 basis points of the transaction value. This figure does not include internal costs or fees paid to consulting firms or investment banks. Due diligence is also significant in terms of the time to completion: it takes an average of 134 days to complete a merger after publicly announcing it (Offenberg and Pirinsky, 2015). Without the terms of a deal in place, potential acquirers will be unwilling to invest the resources needed to complete due diligence in a timely manner.

Providing access to proprietary information may also impose strategic costs on the seller. A strategic bidder may use information gathered during due diligence to become more competitive with the target in the case the deal fails (see e.g., Marquardt and Zur, 2015; Wangerin, 2019). A recent example of this behavior occurred when Urban Outfitters walked away from a deal to acquire the apparel subscription service Le Tote, after months of due diligence during which Urban Outfitter's executives visited Le Tote's warehouses multiple times to gain an understanding of its business model. Subsequently, Urban Outfitters launched its own apparel subscription service, prompting Le Tote to file a lawsuit against Urban Outfitters alleging a breach of the non-disclosure agreement.[3]

The right to conduct due diligence after terms have been negotiated does not preclude due diligence prior to negotiations. Indeed in M&A, a non-trivial portion of the information gathering takes place during a pre-public phase.[4] Lajoux (2010), Marquardt and Zur (2015), and Wangerin (2019) describe the due diligence that is performed during those deals. Figure 1 illustrates this process. After demonstrating interest and signing a confidentiality agreement, bidders performs due diligence using data supplied by the firm. This phase of a deal is known as the pre-public phase.

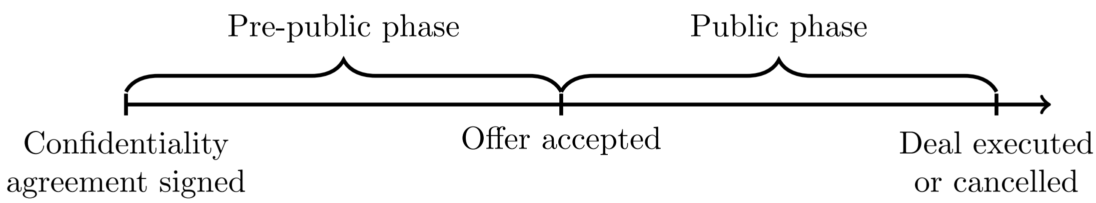


Figure 1. Different phases of an M&A of a company.

After this pre-public phase, the target firm holds an auction or negotiates bilaterally with interested bidders. During the public phase, the acquirer performs additional due diligence by verifying financial information, legal information, existing obligations to suppliers and customers, the state of physical assets, etc. A significant fraction of deals are canceled during the public phase: 1/8 of deals that were publicly announced between 1986 and 2018 were ultimately canceled (Heath and Mitchell, 2022).

[3]See Financial Times article dated June 2, 2020: "Is a M&A NDA really just a shadow non-compete?" (date accessed: August 28, 2020)

[4]See Hansen (2001) and Boone and Mulherin (2007) for discussion of the takeover process in M&A.

Failure to conduct proper due diligence can have severe financial consequences for the acquirer. A recent example is the Wirecard scandal. In 2019, Wirecard issued $1bn worth of convertible debt to SoftBank, which it then sold to investors. A year later Wirecard went bankrupt rendering the debt worthless. An undetected accounting fraud—a non-existing cash account—had left a €1.9bn hole the company's balance sheet.[5] Another example is Hewlett-Packard's acquisition of Autonomy in 2011. One year after the acquisition, Hewlett-Packard wrote down $8.8bn in the value of the $11bn acquisition after accounting irregularities that predated the acquisition were discovered. Hewlett-Packard said that no red flags were raised during due diligence.[6]

Due diligence is also pervasive in private equity and real estate. In private equity, due diligence involves a significant amount of information production because the target has not been subject to the same public scrutiny or disclosure requirements as public firms. According to data from PitchBook, over 40% of completed private equity deals took more than 15 weeks to close. In residential real-estate transactions, the purchaser typically retains the option to terminate the contract pending the review of seller disclosures, surveys, and inspections, all of which are forms of due diligence. Commercial real estate deals almost always involve an extended due diligence phase that can last for months. The typical due diligence checklist for commercial real estate transactions includes title and zoning verification, tenant and lease matters, existing and potential legal claims, insurance claims, and physical property inspection (Brueggeman and Fisher, 2019, chap. 13).

B. *Related Literature*

Our work relates to the literature on auctions of real options pioneered by Board (2007) and Cong (2018, 2020). One of our contributions is to elucidate that the ability to conduct due diligence is akin to a having a real option and thus, the real options framework is

[5]See Financial Times article dated June 22, 2020: "SoftBank executives set to lose profits from Wirecard trade" (date accessed: August 24, 2020).

[6]See Financial Times article dated November 20, 2012: "HP takes $8.8bn hit over Autonomy" (date accessed: August 24, 2020)

a useful lens through which to view the role of due diligence. This conceptual point, while not complex or technically difficult to formalize, has not been made in the existing literature and we believe will be a useful insight for applied work on the topic.

Of course, there are important economic differences between a firm selling itself to an acquirer and the government auctioning off the rights to drill oil on a tract of land. Auctions have formal rules (e.g., a first-price auction with sealed bids), and bidders' identities are procedurally irrelevant owing to anonymity and/or the imposition of symmetry under the formal rules. Whereas, in corporate acquisitions, the identity and attributes of potential acquirers is known to the target, and auctions are informal. That is, the target firm may consider aspects of the offer other than the price (including the characteristics of the bidder).

Naturally, this leads to differences between our theoretical framework and those in the existing literature on selling options. To be concrete, the existing literature focuses on settings where bidders are privately informed and the primary consideration of the seller is to minimize bidders' information rent. In our setting, bidder valuations are commonly known (corresponding to their lack on anonymity) and we pose the game as an informal auction. Moreover, because due diligence is (at least) partly to overcome adverse selection (see the discussion in the previous subsection), we extend our baseline model to a setting with a privately informed seller, which is arguably more relevant in corporate acquisitions than in traditional auctions.

DeMarzo et al. (2005) and Gorbenko and Malenko (2011) analyze auctions where the payments to the seller are contingent on the winning bidder's private information, whereas in our model the transfer depends on the acquirer's execution decision. There is also a literature on information acquisition in auctions prior to bidding (Matthews, 1984; Stegeman, 1996; Persico, 2000; Shi, 2012), whereas information is acquired after bidding in our model .

We also contribute to the theoretical literature on M&A.[7] Within this space, our paper is most closely related to the literature that studies the takeover (auction) mechanism,

[7] See Betton et al. (2008) and Eckbo et al. (2020) for excellent reviews of this literature.

see Bulow and Klemperer (1996), Hansen (2001), Ye (2007), Quint and Hendricks (2018), and Gorbenko and Malenko (2018, 2019). Our contribution is in studying the impact of the due diligence during the public phase, which is especially relevant in takeover auctions (Marquardt and Zur, 2015; Wangerin, 2019).

Our work also relates to the literature on product returns and refund policies (Davis et al., 1995; Courty and Hao, 2000; Matthews and Persico, 2007; Inderst and Ottaviani, 2013; Krähmer and Strausz, 2015). This literature explores settings in which a customer can learn from using the product and can decide whether to return it after purchasing. Our paper differs in two ways from this literature. First, the information acquisition process is dynamic in our paper, whereas it is either exogenous or determined ex-ante in this literature. Second, the transfer of ownership occurs after information acquisition (if at all) in our paper, meaning the acquirer obtains a call option once the agreement is in place. Whereas in the product returns literature, ownership is transferred prior to the right to return the good endows the buyer with a put option, which lead to fundamentally different incentives in the exercise decision.

There is a key timing difference in this paper compared to earlier work by Daley and Green (2012, 2020) where learning takes place before the buyer and seller have agreed to terms and ends as soon as the seller accepts an offer. In this paper, the buyer has the option to continue to learn by performing due diligence after terms of the deal have been set. The difference in timing leads to fundamentally different predictions. Perhaps most notable among them is that inefficient delays can occur even with symmetric information and common knowledge of gains from trade (see Section II.A). Whereas if learning ends once an offer is accepted, trade is both fully efficient and immediate with symmetric information and common knowledge of gains from trade.

The remainder of the paper is organized as follows. In Section I, we present the baseline model. Section II studies the timing of due diligence. Section III extends the analysis to heterogeneous bidders. Section IV considers unconditional transfers, break-up fees and other costs associated with delay. Section V presents quantitative implications. Section VI concludes. All proofs are relegated to the Appendix or Internet Appendix.

I. Baseline Model

There is one seller and multiple competitive bidders. The seller owns an asset, which is either of high or low value (also referred to as type), denoted by $\theta \in \Theta = \{L, H\}$. Asset type is unknown to all players. Bidders have a common value for the asset, V_θ, and the seller's reservation value for the asset is k.[8] We assume there are gains from trade if and only if the asset is high value, $V_H > k > V_L$, and normalize $V_L = 0$. All agents are risk-neutral and discount future cash flows at rate r.

The model takes place in continuous time. At time $t = 0$, the seller holds an informal auction for the asset. Each bidder makes a price offer and the seller selects which bid (if any) to accept. After the auction, the winning bidder (henceforth, the *acquirer*) can perform due diligence and decides when (if ever) to execute the transaction and complete the deal. During due diligence, the acquirer gathers information about the type of the asset. If the winning bid is P and the acquirer executes the transaction at date τ, then the seller's (net) payoff is $e^{-r\tau}(P - k)$ and the acquirer's payoff is $e^{-r\tau}(V_\theta - P)$. If the transaction is never executed then all players' payoffs are zero.[9]

A. Learning

The seller and bidders have a common prior $q_0 \in (0, 1)$, which is the probability they assign to $\theta = H$. During due diligence, both parties observe information about the asset's type from a Brownian information process

$$dX_t = 1_{\{\theta=H\}}dt + \frac{1}{\phi}dB_t, \tag{1}$$

[8]The assumption of common values in the baseline model is innocuous. The results in this section remain unchanged if bidders have private values that are ex-ante unknown and identically distributed.

[9]For pedagogical reasons, we focus on this simple class of contracts in the baseline model. We explore enrichments to the contracting space in Section IV.

where B_t is a standard Brownian motion on the canonical probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and ϕ is the signal-to-noise ratio or the "speed" of learning during due diligence.[10] A higher ϕ makes the information process less noisy and therefore more informative about θ per unit of time. The information process $\{X_t\}_{t \geq 0}$ generates a filtration $(\mathcal{H}_t)_{t \geq 0}$. Let q_t denote the probability the acquirer assigns to $\theta = H$ conditional on information acquired up to date t. For computing q_t, X_t is a sufficient statistic for the entire path.[11] Therefore, q_t can be computed from Bayes' rule as

$$q_t = \frac{q_0 f_t^H(X_t)}{q_0 f_t^H(X_t) + (1 - q_0) f_t^L(X_t)},$$

where f_t^θ denotes the pdf of X_t conditional on θ. Using Ito's lemma, the evolution of q_t is given by $dq_t = \phi^2 q_t(1 - q_t)(dX_t - q_t dt)$. Notice that $\phi(dX_t - q_t dt)$ is the increment of a standard Brownian motion on the probability space $(\Omega \times \Theta, \mathcal{F} \times 2^\Theta, \mathcal{P} \times \upsilon)$, where υ is the measure over Θ defined implicitly by q_0.

B. *Strategies and Equilibrium Concept*

The game can be divided into two stages. In the first stage, bidders simultaneously make offers to the seller and the seller decides which offer to accept. In the second stage, the acquirer conducts due diligence until she deems it optimal to execute the transaction. The second stage is a proper subgame, which we refer to as the *due diligence subgame.*

Suppose the winning bid price is P. In the due diligence subgame with initial belief q, the acquirer chooses a stopping time to maximize her expected discounted payoff

$$F_B(q|P) = \sup_\tau \mathbb{E}_q \left[e^{-r\tau}(V(q_\tau) - P) \right],$$

[10] Whether or not the seller observes X_t is irrelevant for all of our results except those pertaining to the timing of due diligence (i.e., Propositions 5 and 7).

[11] This statement follows from Girsanov's theorem upon observing that the Radon-Nikodym derivative for a change in the measure (over paths) conditional on $\theta = H$ to the measure conditional on $\theta = L$ depends only on X_t.

where $V(q) \equiv qV_H + (1-q)V_L$. The acquirer's strategy in the due diligence subgame is therefore a collection of stopping times, which are indexed by the winning bid price with elements denoted by $\tau(P)$.

The seller must take into account the acquirer's strategy in the due diligence subgame when deciding which offer to accept. Let $F_S(q|P)$ denote the seller's expected payoff from accepting an offer of P, starting from belief q:

$$F_S(q|P) = \mathbb{E}_q\left[e^{-r\tau(P)}(P-k)\right].$$

Clearly, the seller will reject any offer below k and the bidder never completes the transaction if the price is higher than V_H. Therefore, we can restrict attention to bids in the interval $[k, V_H]$. Our equilibrium concept is subgame perfect Nash equilibrium, henceforth referred to simply as *equilibrium*.[12]

C. Equilibrium Analysis

We solve for the equilibrium by backward induction. Given any price P, the acquirer faces a stopping problem of when to complete the transaction. The solution to this problem is to complete the transaction as soon as the belief exceeds a threshold.

LEMMA 1 (Acquirer-Optimal Execution): *Given any price $P \in [k, V_H]$, the acquirer completes the deals as soon as q_t exceeds*

$$b(P) = \frac{1}{1 + \frac{1-\underline{q}(P)}{\underline{q}(P)} \times \frac{u-1}{u}} > \underline{q}(P), \tag{2}$$

where $u = \frac{1}{2}(1+\sqrt{1+8r/\phi^2})$ and $\underline{q}(P) = \frac{P-V_L}{V_H-V_L}$. That is, $\tau^*(P) = \inf\{t > 0 | q_t \geq b(P)\}$.

The optimal acquisition threshold depends on the product of two terms. The first term depends only on the belief such that the expected value of the asset is equal to the price, $\underline{q}(P)$, which is increasing in P. This belief is akin to the strike price on a call

[12]The role of subgame perfection is to require the acquirer to play optimally in the due diligence subgame for any possible winning bid and not just the bid that is accepted on the equilibrium path.

option: for beliefs above $\underline{q}(P)$ the option is in the money. The second term depends only on $\gamma \equiv \phi^2/r$, which is the speed of learning per unit cost of time. The higher is γ, the greater is the option value from due diligence and the higher is the execution threshold. As $\gamma \to 0$, the value of conducting due diligence goes to zero and $b(P) \to \underline{q}(P)$. Figure 2 illustrates the acquirer's value function, the optimal execution threshold, and its relation to $\underline{q}(P)$.

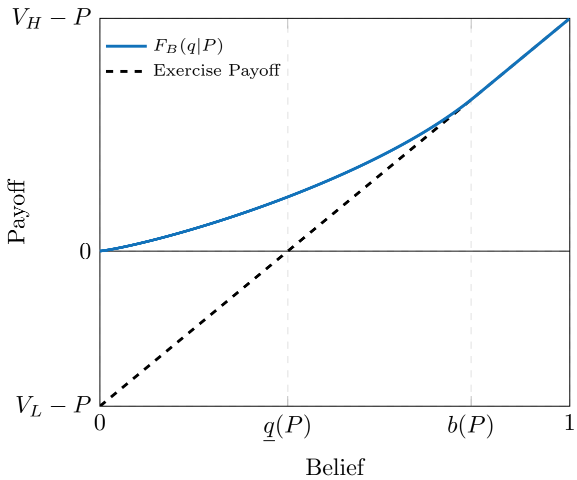


Figure 2. Solution to the acquirer's problem in the due diligence subgame.

Having characterized the acquirer's strategy in the due diligence subgame, consider the first stage wherein the price is determined. Because bidders are identical and perfectly competitive, the equilibrium price maximizes the seller's expected payoff. Clearly, a higher price is good for the seller conditional on execution. However, because a higher price implies a higher execution threshold, it also implies more delay and a higher chance of deal failure. When the price is low (i.e., close to the seller's reservation value), the marginal cost of a higher price is relatively small and outweighed by the higher payoff conditional on execution. For higher prices, the marginal cost of a higher price outweighs the marginal benefit. As a result, the seller's payoff is initially increasing in P and eventually decreasing in P.

To characterize the seller-optimal price, let $P_0(q)$ denote the highest price at which the acquirer is willing to forego due diligence and execute the transaction immediately given the belief is q. In order to induce immediate execution, the seller must provide some rents to the acquirer. Thus, $P_0(q) < V(q)$. A closed-form expression for P_0 can be obtained by inverting $b(P)$ from equation (2) to get

$$P_0(q) = \frac{(1-q)uV_L + q(u-1)V_H}{u-q}.$$

Next, consider the hypothetical stopping problem in which the seller chooses when to accept $P_0(q_t)$:

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau} (P_0(q_\tau) - k) \right]. \tag{HSP}$$

The payoff from the solution to this problem provides an upper bound on the seller's equilibrium payoffs. The solution necessarily involves a belief threshold above which it is optimal to stop. The following assumption says that it is never optimal to stop below the threshold.

ASSUMPTION 1: *There exists a* $q^* \in (0,1)$ *such that the solution to* (HSP) *is* $\tau_{HSP} = \inf\{t \geq 0 : q_t \geq q^*\}$.

Assumption 1 provides an intuitive sufficient condition for the equilibrium to have a single execution threshold. When Assumption 1 fails, there may exist multiple disjoint intervals over which the deal is executed. Assumption 1 is not directly about primitives of the model. The next lemma provides sufficient conditions on primitives for it to hold.

LEMMA 2: *Fixing all other parameters, Assumption 1 is satisfied if either:*

(i) $k \geq \bar{k}$ *for some* $\bar{k} \in (V_L, V_H)$.

(ii) $\gamma > \bar{\gamma}$ *for some* $\bar{\gamma} > 0$.

Assumption 1 can fail if k is small enough and γ is neither too large, nor too small. Our numerical analysis suggests the region of the parameter space in which Assumption 1 can fail is rather small: $\bar{k}$ is less than 5% of V_H across all γ.[13] However, it necessarily fails if there is common knowledge of gains from trade (i.e., $k < V_L$). We analyze this case in Section II.A, but assume that Assumption 1 holds until then.

[13]Our proof of Lemma 2 establishes that $\bar{k}$ depends only on γ and V_H.

LEMMA 3 (Seller-Optimal Price): *Let $P^* \equiv P_0(q^*)$. The seller optimal price, denoted $P_S(q_0)$, is given by*

$$P_S(q_0) = \begin{cases} P^* & \text{if } q_0 \leq b(P^*) \\ P_0(q) & \text{if } q_0 > b(P^*) \end{cases}$$

For $q_0 \geq b(P^*)$, the seller prefers an offer that induces the acquirer to forego due diligence and execute the transaction immediately. For $q_0 < b(P^*)$, the seller prefers the acquirer to conduct due diligence with the hope of transacting at price P^* in the future rather than settle for $P_0(q_0)$ today. Naturally, competition among bidders drives the winning offer to the seller's preferred price.

PROPOSITION 1: *There exists a unique equilibrium outcome. In it,*

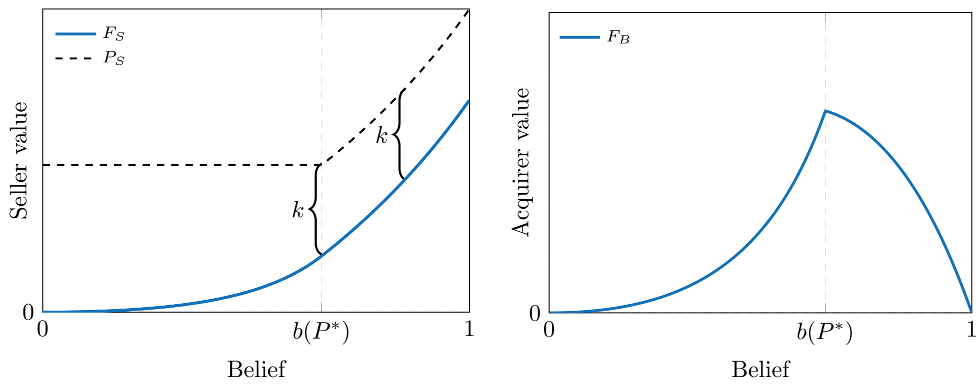
(i) The winning offer is the seller-optimal price, $P_S(q_0)$.

(ii) In the due diligence subgame, the acquirer plays according to $\tau^*(P)$.

(iii) There is a period of due diligence if and only if $q_0 < b(P^*)$.

Figure 3 illustrates the equilibrium payoffs and prices as they depend on the initial belief. While the seller's payoff is increasing in the belief , the acquirer's value is maximized at $b(P^*)$. Notice from Figure 3A that the seller-optimal price is constant for all $q_0 < b(P^*)$, corresponding to the region in which due diligence takes place. As a result, the acquirer's payoff is increasing in this region, but decreasing above $b(P^*)$ where the price increase offsets the increase in the gains from trade (Figure 3B). Moreover, because the price is constant in the due diligence region, the seller's equilibrium payoff is the same as the payoff in (HSP). This payoff equivalence has two important implications.

First, play is renegotiation proof along the equilibrium path—the seller does not have an incentive to try to renegotiate the price in response to information being revealed. Second, whether due diligence takes place before, after, or during bidding is payoff irrelevant (we formally demonstrate this result in Section II). If bidders can conduct due diligence prior to or during negotiations, then the option to conduct further due diligence



Panel A. Equilibrium price and seller value

Panel B. Acquirer value

Figure 3. Equilibrium prices and payoffs as a function of the belief.

after an offer is accepted may not be exercised. However, the equilibrium payoffs remain unchanged.

D. Implications

How does the acquirer's ability to conduct due diligence affect equilibrium quantities? Does due diligence improve efficiency or lead to unnecessary delays? To answer these questions, we first utilize the Martingale property of the belief process to derive expressions for quantities of interest. We then consider the effect of an increase in ϕ and compare the equilibrium outcome to two benchmarks.

LEMMA 4 (Deal Completion and Time to Completion): *Suppose that $q_0 < b(P^*)$. Let τ^* denote the time at which the deal is completed.*

(i) The probability that the deal is eventually completed is $\mathbb{P}(\tau^* < \infty) = \frac{q_0}{b(P^*)}$,

(ii) Conditional on the deal being completed, the expected time to completion is

$$\mathbb{E}\left[\tau^* | \tau^* < \infty\right] = \frac{2}{\phi^2}\left(\ln\left(\frac{b(P^*)}{1 - b(P^*)}\right) - \ln\left(\frac{q_0}{1 - q_0}\right)\right).$$

Speed of Learning Recall that ϕ denotes the signal-to-noise ratio and measures the "speed" with which the acquirer learns during due diligence. The following proposition summarizes how ϕ affects equilibrium quantities.

PROPOSITION 2 (Speed of Learning): *An increase in the acquirer's speed of learning from* ϕ_1 *to* ϕ_2 *increases the execution threshold,* $b_2 > b_1$*, and*

(i) The equilibrium price increases for all $q_0 < \hat{q}$ *and decreases for all* $q_0 > \hat{q}$*, for some* $\hat{q} \in (b_1, b_2)$*;*

(ii) The probability of deal completion decreases for all $q_0 < b_2$*, and is unchanged for all* $q_0 \geq b_2$*;*

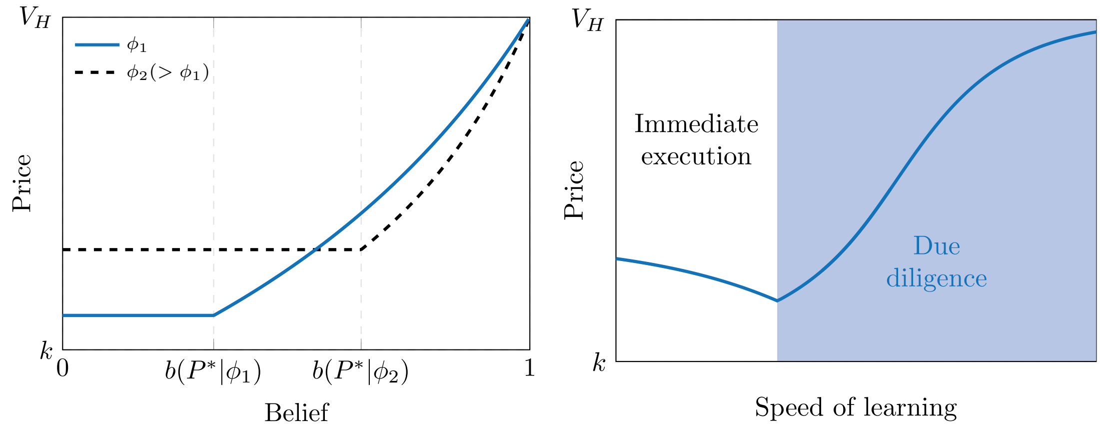
(iii) The conditional expected time to completion decreases for all $q_0 < q'$*, increases for all* $q_0 \in (q', b_2)$*, and is unchanged for all* $q_0 \geq b_2$*, for some* $q' \in (0, b_1)$*.*

Increasing the speed of learning has two opposing effects on the equilibrium price. To decompose them, note that the equilibrium price can be expressed as $P_S(q) = \max\{P^*, P_0(q)\}$, where recall $P_0(q)$ is the highest price at which the acquirer is willing to execute the transaction immediately given the belief is q.

Holding the function $P_0(\cdot)$ fixed, increasing the speed of learning makes the seller prefer a higher execution threshold (and thus a higher price), as it would be reached more quickly. As a result of this force, P^* is increasing in ϕ. On the other hand, increasing the speed of learning confers greater option value to the acquirer, meaning $P_0(q)$ is decreasing in ϕ. For low beliefs, the price is determined by P^*, so the first force dominates. For high beliefs, the price is determined by P_0, so the second force dominates. Figure 4(a) illustrates.

An implication of this result is that for a fixed belief $q > \underline{q}(k)$, the price is U-shaped in ϕ as in Figure 4(b). If learning is slow, then $q > b(P^*|\phi)$ and there is direct execution at $P_0(q)$. As learning speed increases initially, $P_S(q) = P_0(q)$ falls reflecting the acquirer's greater option value, but $b(P^*|\phi)$ increases. Eventually, $b(P^*|\phi)$ reaches q and $P_S(q)$ switches to P^*, which increases with ϕ reflecting the seller's preference for a higher price knowing that learning during due diligence now happens faster.[14] As learning becomes arbitrarily fast, the price limits to V_H and $b(P^*)$ goes to one: the acquirer effectively

[14]If $q \leq \underline{q}(k)$, then $q \leq b(P^*|\phi)$ for all ϕ and the equilibrium price is strictly increasing in ϕ.



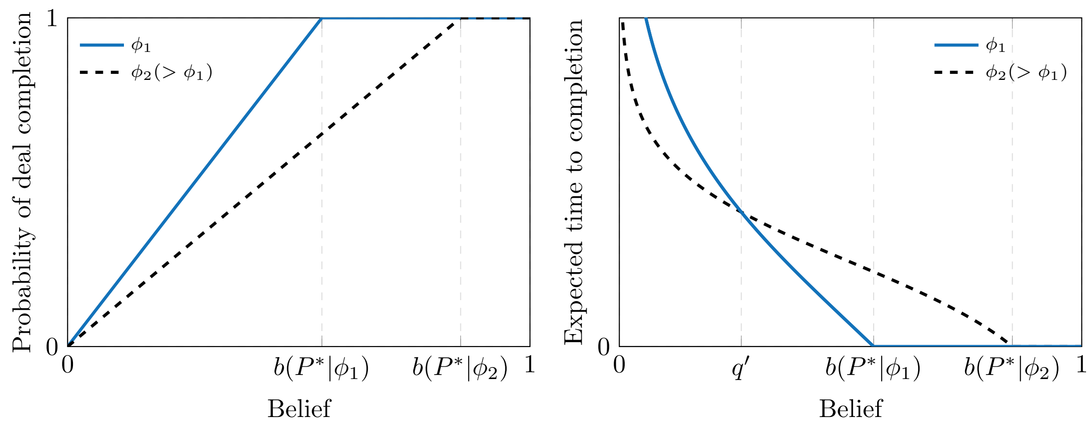
Panel A. Price as it depends on belief

Panel B. Price as it depends on learning speed

Figure 4. Impact of learning speed on equilibrium price.

learns the asset quality instantaneously and executes only if the quality is high, so the best price for the seller is V_H.

Figure 5 illustrates parts (ii) and (iii) of Proposition 2. Increasing ϕ has two effects on completion time. First, $b(P^*)$ increases, which leads to longer time to completion. Second, learning occurs faster, which shortens time to completion. The second effect becomes stronger the longer is the time to completion. Therefore, the first effect dominates for beliefs near the execution threshold, while the second effect dominates for beliefs further away from the threshold.



Panel A. Probability of deal completion

Panel B. Conditional expected time to completion

Figure 5. Impact of learning speed on probability of deal completion and time to completion.

Proposition 2 has empirical implications, which could be tested either by developing proxies for ϕ or by using the deal premium as a proxy for ϕ while controlling for value parameters. To illustrate the latter approach, note that the deal premium (i.e., $P^* - k$) is increasing in ϕ for transactions that involve due diligence. Thus, Proposition 2 predicts that, holding the value parameters fixed, deals with a higher premium will be more likely to fail. Moreover, conditional on the deal being completed, the time to completion is non-monotone in the premium. In particular, among deals that are completed relatively quickly (i.e., $q_0 > q'$), a higher premium should be associated with a longer time to completion. However, among deals that take a relatively long time to complete (i.e., $q_0 < q'$), a higher premium should be associated with a shorter time to completion.

Social Optimum Consider a social planner who does not know the asset's type, but has the ability to learn by observing the information generated during the due diligence. The planner's problem is to choose a stopping time to solve

$$\sup_{\tau} \mathbb{E}_q \left[e^{-r\tau} (V(q_\tau) - k) \right].$$

Notice that the planner's problem is identical to the acquirer's problem except that the strike price of the option is the seller's reservation value rather than the winning bid.

PROPOSITION 3 (Social Optimum): *The socially optimal execution threshold is $b(k)$, which is strictly less than $b(P^*)$. Therefore, in equilibrium, the acquirer conducts "too much" due diligence and the probability of deal failure is too high relative to the social optimum.*

Intuitively, this result derives from the fact that the acquirer only captures part of the surplus, and therefore does not internalize the full cost of delay and deal failure. To induce efficient execution, the price paid conditional on executing the transaction should be set equal to k. But if the price is k, then the seller captures none of the surplus. Competition among bidders drives the price above k, which in turn leads to inefficient execution.

No Due Diligence Consider a second benchmark without due diligence: if the seller accepts the bidder's offer then the transaction is immediately executed. In this case, the seller's expected payoff is increasing in the price. As a result, competitive bidders offer a price equal to the expected asset value and the seller accepts the offer when it is above his reservation value.[15] Without the ability to conduct due diligence, the acquirer makes zero profit. Therefore, the acquirer's ability to perform due diligence allows her to extract positive surplus. Note that this result holds regardless of whether the acquirer actually conducts any due diligence along the equilibrium path. The mere option to conduct due diligence is what facilitates surplus extraction.

Figure 6 plots the social surplus and seller value both with and without due diligence. The figure suggests that due diligence improves surplus for low beliefs but hinders it for intermediate beliefs. The following proposition formalizes this result.

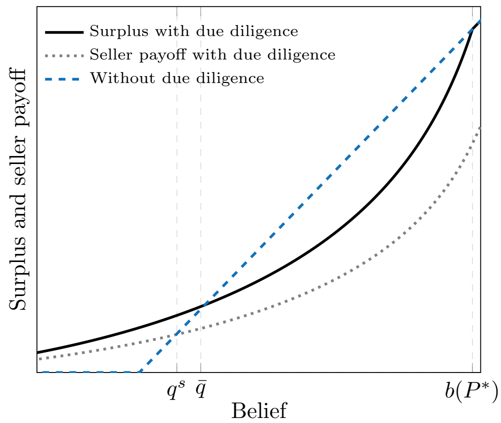


Figure 6. Impact of due diligence on surplus. Note that without due diligence, the total surplus and the seller's payoff coincide. The belief q^s is the threshold below which the seller prefers due diligence over the no due diligence benchmark. The belief $\bar{q}$ is the threshold below which due diligence increases social surplus as formalized in Proposition 4.

PROPOSITION 4 (Social Surplus): *There exists a belief $\bar{q} < b(P^*)$ such that*

(i) For $q_0 < \bar{q}$, due diligence increases social surplus.

(ii) For $q_0 \in (\bar{q}, b(P^*))$, due diligence decreases social surplus.

[15]This outcome also obtains in the limit as $\gamma \to 0$.

(iii) For $q_0 \geq b(P^*)$, due diligence does not change social surplus.

Intuitively, allowing for due diligence increases surplus when the deal would necessarily fail without it. However, because the acquirer is too diligent relative to the social optimum (Proposition 3), allowing for due diligence when it is socially optimal to execute decreases surplus.

II. The Timing of Due Diligence

In practice, some amount of investigation may take place before or during negotiations between bidders and the seller. In this section, we explore the timing of due diligence. We first extend the model to allow for due diligence before or during bidding. We demonstrate a timing irrelevance result, which shows that our results are unchanged under this alternative specification. In other words, it is the acquirer's *option* to conduct (more) due diligence after her offer is accepted that is important for our results, not the precise timing of when this learning takes place. We then characterize the equilibrium of the model when there is common knowledge of gains from trade (e.g., due to synergies), demonstrate the relevance of timing therein, and discuss the implications.

The game again consists of two phases. However, the first stage is now dynamic, with competitive bidders conducting due diligence while simultaneously making offers. To facilitate comparison with the baseline model, we assume that all players observe the same information process in the first stage (as given in (1)). Let ν be the time at which the seller accepts an offer and denote the offer by P_ν. After the seller accepts an offer, the due diligence subgame ensues: the acquirer can continue to perform due diligence and decide when if ever to execute the transaction at the price P_ν. Applied to corporate takeover auctions, one can interpret bids made prior to ν as indicative bids (Ye, 2007; Quint and Hendricks, 2018; Liu and Officer, 2019) made during the "pre-public" phase of the negotiation (see Subsection A in the introduction). Whereas at date ν, the seller solicits formal bids.

The main insight of this section is that allowing for due diligence prior to or during bidding does not substantively alter the model's predictions.[16]

PROPOSITION 5 (Timing Irrelevance): *In any equilibrium with dynamic bidding, the payoffs of the seller and the acquirer are the same as in the equilibrium of the baseline model.*

Recall that under Assumption 1, the seller's equilibrium payoff in the baseline model is the same as in the solution to the hypothetical stopping problem of when to accept $P_0(q_t)$ (see (HSP)). With dynamic bidding, the seller can do no worse: by definition, bidders are willing to offer $P_0(q_t)$ and execute immediately. The interesting part is that the seller can also do no better. This result follows from the seller-optimal price being independent of the prior in the due diligence region. Regardless of what information is uncovered during due diligence, the seller has no desire to renegotiate the price. Hence, he has nothing to gain by waiting to accept an offer.

For $q < b(P^*)$, the price offers and the acceptance time are not uniquely determined with dynamic bidding. In one equilibrium, bidders offer $P_0(q_t)$ at all t and $\nu = \tau(P^*)$, meaning that due diligence is conducted only *prior* to reaching an agreement and never *after* the seller accepts. In another equilibrium, bidders offer $P_S(q_t)$ at all t, $\nu = 0$, and due diligence only takes place after the seller accepts (as we assume in the baseline model). However, in any equilibrium the execution price and execution time are P^* and $\tau(P^*)$ respectively. Hence, both the seller and the acquirer's payoffs are identical.

Naturally, there are also considerations outside of the model that may render the timing decision relevant. As discussed in Subsection A in the introduction, providing access to proprietary information may also impose strategic costs on the seller. Further, multiple bidders analyzing the same information may involve inefficient duplication of information processing costs. Both of these considerations discourage the seller from using dynamic bidding. On the other hand, if bidders have private values then dynamic bidding allows the seller to capture some of the option value from ensuring the highest bidder wins.

[16]This finding is in contrast to Cong (2020), where the seller may optimally choose to (inefficiently) delay the date of the auction in order to reduce privately informed bidders' information rents.

Assumption 1 is sufficient to ensure timing irrelevance. It is also necessary in the sense that if it fails, then there exists priors such that the seller prefers dynamic bidding. However, rather than explore this relatively small region of the parameter space (see the discussion following Lemma 2), we extend the parameter space to allow for common knowledge of gains from trade (i.e., $k < V_L$), which necessarily violates Assumption 1.

A. *Synergies and Common Knowledge of Gains from Trade*

We have assumed that the efficient allocation of the asset is type dependent ($V_L < k < V_H$). If instead $k < V_L$, then there is common knowledge of gains from trade (CKGT). CKGT is more likely to arise in strategic acquisitions, where the primary motivation for the bidder to acquire the target is due to synergies. When the synergies are sufficiently large, there are gains from trade with the target regardless of the value of the firm's existing assets. Whereas a financial bidder is typically only interested in acquiring targets that are undervalued.[17]

With CKGT, it is socially optimal to execute the transaction immediately for all prior beliefs. We have already seen the equilibrium without CKGT features excessive due diligence. Under what conditions does a similar result obtain with CKGT? The answer is provided by the following proposition.

PROPOSITION 6 (Equilibrium with CKGT): *Suppose that $k < V_L < V_H$. Then, there exists a $\underline{\gamma} > 0$ such that the equilibrium with static bidding is as follows.*

(i) If $\gamma > \underline{\gamma}$, there exist two thresholds $q_a < q_b$ such that

(a) For $q_0 \in (q_a, q_b)$, the winning offer is $P_0(q_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(q_b))$.

(b) For $q_0 \notin (q_a, q_b)$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

[17] This difference in the motivations of strategic versus financial bidders is discussed in Gorbenko and Malenko (2014) and can explain their finding that strategic bidders have (on average) higher value for targets than financial bidders.

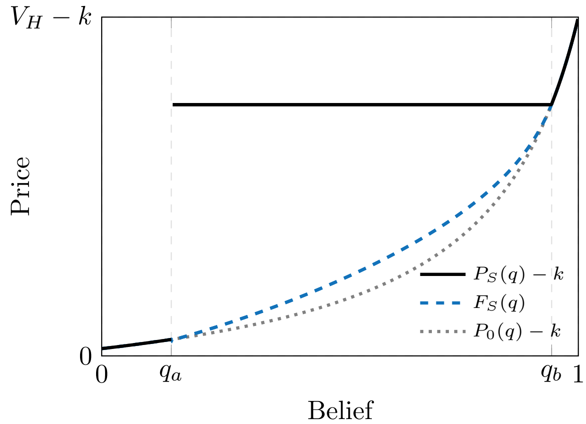


Figure 7. Equilibrium price and the seller's payoff with CKGT.

(ii) If $\gamma \leq \underline{\gamma}$, the winning offer is $P_0(q_0)$ and the transaction is executed immediately.

When the speed of learning relative to the cost of delay is below $\underline{\gamma}$, due diligence is too costly to appeal to the seller. More interestingly, when γ is above $\underline{\gamma}$, due diligence once again emerges. For intermediate beliefs, the seller prefers a price higher than the one that would induce the acquirer to execute immediately. Thus, even when there is no social motive for due diligence, the seller can benefit from inducing the acquirer to conduct due diligence as a means of reducing the acquirer's rents. Figure 7 illustrates the equilibrium price and seller's payoff in this case ($\gamma > \underline{\gamma}$). Compared to Figure 3A, the key difference is left-most interval (i.e., $[0, q_a]$) in which the equilibrium features immediate execution and the price drops to $P_0(q)$ to induce it. That is, CKGT can lead to two execution regions as opposed to the single execution region when (HSP) holds.[18]

One novel implication of the equilibrium structure in Proposition 6 is that dynamic bidding is no longer payoff equivalent to static bidding.

PROPOSITION 7 (Timing Relevance with CKGT): *Suppose that $k < V_L$. Then, there exists a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$, the seller strictly prefers dynamic bidding. Moreover, with dynamic bidding, all due diligence takes place prior to the seller accepting an offer.*

To understand why the seller prefers dynamic bidding, suppose that the initial prior

[18] When Assumption 1 fails and $k \geq V_L$, the equilibrium with static bidding is similar to Proposition 6, part (i), with the addition of a third belief threshold $q_c \leq q_a$ such that for $q_0 < q_c$, the offer is $P_0(q_c)$ and the acquirer conducts due diligence until $\tau^*(q_c)$.

is $q_0 \in (q_a, q_b)$ and thus the winning offer with static bidding is $P_0(q_b)$. If bad news is revealed during due diligence such that the belief drops below q_a, then the seller optimal price also drops (see Figure 7). The seller would therefore like to renegotiate the price down to induce the acquirer to execute immediately, which is not feasible under static bidding.[19] However, with dynamic bidding, the seller can accomplish the same outcome by delaying acceptance until due diligence has been effectively completed (i.e., the belief reaches either q_a or q_b) prior to accepting an offer.

Proposition 7 has implications for whether due diligence takes place before or after the seller and acquirer agree to terms. Our model suggests that in strategic acquisitions (i.e., where the synergies are sufficiently large that CKGT holds), there is an advantage to conducting due diligence prior to terms being reached (e.g., in the "pre-public" phase), whereas there is no such advantage for purely financial transactions. Thus, our results suggest that more of the due diligence will take place in the "pre-public" phase for deals with strategic bidders compared to those with financial bidders. Of course, providing multiple strategic bidders with access to sensitive information about the firm can impose significant costs, non-disclosure agreements notwithstanding. Another prediction relates to the likelihood of deal completion. With dynamic bidding and CKGT, the deal is completed with probability one. A testable implication of this line of argument is that deals with financial acquirers should be more likely to fail compared to deals with strategic ones.

III. Heterogeneous Bidders

In this section, we extend the model to allow for heterogeneity in bidder valuations. More specifically, we introduce a *strong* bidder who has a higher valuation for the asset than the other bidder, who we will now refer to as the *weak* bidder.[20] To build on the

[19] Moreover, if the acquirer were allowed to renegotiate, the seller would be subjected to a hold-up problem.

[20] The number of weak bidders is inconsequential provided there is at least one. If there are multiple strong bidders then the results from the baseline model apply.

notation from previous sections, we let all of the objects already defined pertain to the weak bidder and we introduce a superscript st to denote objects pertaining to the strong bidder. That is, the strong bidder's values for the asset are $V_H^{st} > V_H$ and $V_L^{st} = 0$. We maintain the assumption that the bidder's values are common knowledge conditional on θ.

To characterize the equilibrium with bidder heterogeneity, it is useful to explore the nature of the strong bidder's advantage. As in most settings with bidder heterogeneity, a higher valuation means the strong bidder is always willing to match the bid of the weak bidder. The typical result in such settings is then that the strong bidder wins the auction by matching the highest bid the weak bidder is willing to make. What is novel about our setting is that the strong bidder has an additional advantage that she can exploit—her willingness to execute more quickly—as we now describe.

As in Lemma 1, if the strong bidder becomes the acquirer with price P, she will complete the deal as soon as the belief exceeds an optimal threshold, $b^{st}(P)$. Critically, because her expected surplus from deal completion is larger than the weak bidder's, she is more eager to complete the deal.

LEMMA 5: *Fixing any price $P \in [k, V_H^{st}]$, the strong bidder executes the transaction more quickly and with a higher probability. That is, $b^{st}(P) < b(P)$.*

Holding the price fixed, the seller benefits from this expediency: $F_S^{st}(q|P) \geq F_S(q|P)$, where the inequality is strict for all $P \leq V_H$ and $q \in (0, b(P))$. Hence, if both the strong and weak bidders submit the same bid, the seller will accept the strong bidder's offer. However, the strong bidder can do better by shading her bid downward and relying on the seller's preference for faster execution.

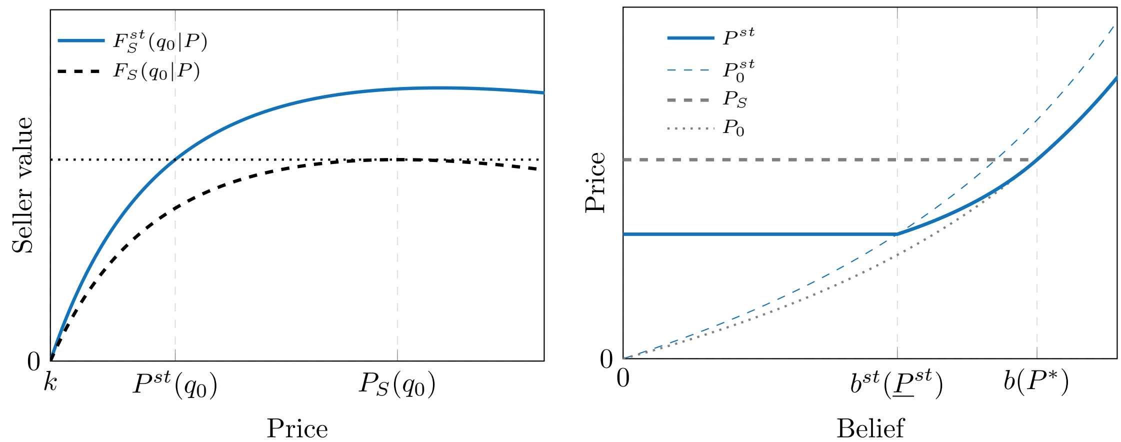
Lemma 3 characterized the most attractive offer that the weak bidder can submit to the seller: $P_S(q_0)$. This remains her equilibrium offer in the presence of a strong bidder. Let $P^{st}(q) \leq P_S(q)$ denote the lowest price such that the seller is indifferent between accepting $P^{st}(q)$ from the strong bidder or $P_S(q)$ from the weak bidder. It is straightforward to show that such a price exists. To illustrate, Figure 8A plots the seller's expected payoff as it depends on the price for each type of bidder and for a fixed prior belief. $P^{st}(q)$ is determined by finding the lowest price, P, such that $F_S^{st}(q|P) = F_S(q|P_S(q))$.

PROPOSITION 8: *There exists a unique equilibrium outcome. In it,*

(i) The winning offer is made by the strong bidder at $P^{st}(q_0) \leq P_S(q_0)$.

(ii) In the due diligence subgame, the strong acquirer plays according to $\tau^{st}(P^{st}(q_0))$.

(iii) There is a period of due diligence if and only if $q_0 < b^{st}(P^{st}(q_0))$.



Panel A. Seller payoff as it depends on bidder and price

Panel B. Equilibrium bids

Figure 8. Heterogeneous bidders.

Figure 8B illustrates how the equilibrium bids depends on the prior belief. This figure can be decomposed into three regions. For low priors q, the weak bidder's offer is constant: $P_S(q) = P^*$. The strong bidder out-competes with an offer of $\underline{P}^{st} \equiv P^{st}(q)$, which is also constant in this region of prior beliefs. Conditions analogous to those in Lemma 2 ensure this property (i.e., k or γ large enough—see Lemma A.6 in the Appendix), which we maintain for the remainder of this section. In this low-prior region, the strong bidder conducts due diligence and only executes the deal when her posterior reaches $b^{st}(\underline{P}^{st})$.

At $b^{st}(\underline{P}^{st})$, the strong bidder's offer function, P^{st}, equals her immediate execution price function, denoted by P_0^{st}. Hence, for intermediate priors, the strong bidder optimally competes by making an offer that induces her to execute immediately, while the weak bidder's offer remains implicitly contingent on subsequent due diligence. For priors above

$b(P^*)$, the weak bidder makes an offer that she will execute immediately. In this region, the strong bidder cannot exploit her speed advantage and therefore simply matches the weak bidder's offer.

A notable feature of the equilibrium is that the seller does not accept the highest price offer when the prior is below $b(P^*)$. For intermediate priors, the logic is clear: the strong bidder's offer is a sure thing whereas there are delays and a risk of deal failure associated with the weak bidder's (higher) offer. Yet even when both bids involve subsequent due diligence, the expected delay and risk of failure associated with the strong bidder's offer are smaller, which renders her offer equally attractive despite the lower price.

Our finding, that the seller accepts a lower offer from a stronger bidder, is consistent with a number of empirical facts from markets where due diligence plays an important role. For instance, Hsu (2004) finds that entrepreneurs are willing to accept a 10-14% discount from a venture capital firm with a high reputation. Hsu (2004) posits that high reputation VCs can add more financial value and are thus analogous to the strong bidder in our model, who can increase the value of the start up by more than a weak bidder as long as it is not a lemon. Relatedly, Reher and Valkanov (2021) find that mortgaged homebuyers pay an 11% premium over all-cash buyers. A mortgaged homebuyer is analogous to a weak bidder since their offer is typically contingent on obtaining financing from the bank, which requires additional due diligence (appraisal, title verification, etc.) and the potential for costly delays. Indeed, using a different data set Han and Hong (2022) document that all-cash buyers are associated with a 33% shorter time to close.

Since a shorter time-to-close is valuable to the seller, and not always feasible for the purchaser, residential real estate markets would seem a natural candidate for intermediation. And indeed a number of "iBuyers" that value homes based on algorithms, and make cash offers to the seller, have emerged in recent years. Yet, Buchak et al. (2020) estimate a dynamic structural model and find limited scope for intermediation in residential real estate. This suggests that due diligence plays an important role in real estate markets and foregoing it (as iBuyers do) is unsustainable. Indeed, Zillow permanently closed its iBuying division in late 2021 after writing down more than $540 million in losses from it.

REMARK 1: *With heterogeneous bidders, all formal results from Sections I and II remain valid under suitable updating of notation (e.g., substituting $b^{st}(\underline{P}^{st})$ for $b(P^*)$ in Proposition 4), with the possible exception of Proposition 2(i), which remains unproven.*

A. Comparative Statics

The novel nature of the bidder competition in this model generates perhaps unexpected comparative statics. To begin, consider how bidder valuations affect price. In the baseline model, if the (common) value V_H increases, so too does price. However, if just one bidder's valuation increases (making them the strong bidder with $V_H^{st} > V_H$), this bidder will exploit her execution advantage and win at a price lower than if all bidder had valuations V_H. Indeed, this feature is general: an increase in the strong bidder's valuation V_H^{st} always strengthens her execution advantage, and leads to a *decrease* in the price, as formalized in the following proposition.

PROPOSITION 9 (Effect of Strong Bidder Valuation): *Suppose that $q_0 < b^{st}(\underline{P}^{st})$ (i.e., due diligence will occur). An increase in the strong bidder's valuations, V_H^{st}, leads to a lower price, a lower execution threshold, and higher total surplus in equilibrium.*

Of course, if the strong bidder will immediately execute ($q_0 \geq b^{st}(\underline{P}^{st})$), she is already fully exploiting her execution advantage. In this case, an increase in V_H^{st} has no effect on equilibrium play. Notice also that $b^{st}(\underline{P}^{st})$ is itself decreasing in V_H^{st}: as the strong bidder's valuation rises, she becomes willing to execute immediately (at the price $F_S(P^*|q_0) + k$) for a larger range of priors q_0.

That total equilibrium surplus increases with V_H^{st} is unsurprising. More interesting is the effect of the weak bidder's valuation on efficiency. Notice that the maximum potential surplus does not depend on the weak bidder's valuation, V_H, since the efficient allocation of the asset is either to the strong bidder or to the seller. Nevertheless, V_H does affect the level of surplus attained via equilibrium play.

PROPOSITION 10 (Effect of Weak Bidder Valuation): *Suppose that $q_0 < b^{st}(\underline{P}^{st})$ (i.e., due diligence will occur). An increase in the weak bidder's valuations, V_H, leads to a* *higher price, a higher execution threshold, and a lower total surplus in equilibrium.*

The proposition therefore cuts against the usual intuition that more similar agents should compete more vigorously and thereby generate efficiency gains. Here, instead, stiffening the strong bidder's competition forces them to compete on price and inefficiently slows execution.

IV. Additional Considerations

In this section, we use our framework to discuss additional contractual features and considerations that are not captured in the baseline model. We then explore their quantitative relevance in Section V.

A. Unconditional Transfers

Enriching the contract space to allow for unconditional transfers from the bidder to the acquirer resolves the distortion in the baseline model. We relate this finding to Board (2007) and discuss the limitations of unconditional transfers.

A contract, C, now consists of a pair $C \equiv (U, P)$, where U is an upfront unconditional transfer and P is the price paid contingent on execution. In this case, we obtain the following result.

PROPOSITION 11 (Unconditional Transfers): *With unconditional transfers, the seller optimal mechanism and the winning equilibrium bid involves* $(U, P) = (F_B(q_0|k), k)$ *if* $q_0 < b(k)$ *and immediate execution if* $q_0 \geq b(k)$*. In either case, the acquirer's execution threshold is socially optimal and the seller captures the entire surplus.*

In effect, the acquirer purchases the option to buy the firm at a price of k and, in doing so, transfers all of the surplus to the seller through the unconditional payment. The contingent price is set so that the acquirer correctly internalizes the social cost and benefit of conducting due diligence.

Proposition 11 is related to Board (2007), who studies the sale of a real option in a private value setting. There are two differences. First, Board (2007), assumes that the seller's reservation value evaporates after the option is transferred to the bidder regardless of whether the option is executed, whereas in our setting the seller foregos his reservation value only if the option is executed. Therefore, Board (2007) concludes that the socially optimal mechanism does not involve a contingent transfer. Second, in our model bidders have no private information (so do not earn information rents). Because the seller can extract all of the surplus, there is no tension between revenue maximization and surplus maximization, unlike in Board (2007).

Although allowing for unconditional transfers resolves the distortion in our baseline model, there are a variety of limitations to its use in practical applications. For instance, one feature of the contract in Proposition 11 is that the seller is *indifferent* about whether the deal is executed, meaning that the seller has no incentive to facilitate or cooperate with the due diligence process.

Another likely reason why unconditional transfers are rarely used in practice is asymmetric information. In the Internet Appendix, we extend our baseline model to a setting where the seller is privately informed about the asset value. In that case, unconditional transfers are relatively more attractive to low-value sellers, whereas contingent price offers serve as an effective screening mechanism. Consequently, even with the ability to make unconditional transfers, the equilibrium contract involves contingent prices that are larger than is socially optimal, again resulting in "too much" due diligence.

There are still other limitations to the use of unconditional transfers in practice. Financial constraints limit an acquirer's ability to credibly offer such a payment. If such transfers were used, an agency conflict between the acquiring firm's manager and its investors emerges (e.g., the manager of the acquiring firm and the target could divide the transfer amongst themselves and declare the due diligence process as failed). Finally, if unconditional transfers were common practice, there would be an incentive to create a "shill" firms and collect the transfer even if due diligence was likely to uncover the fraud. All of these reasons likely contribute to the explanation for why unconditional transfers, while common in many auctions, are rarely observed in corporate acquisitions.

B. Delay Costs

In our baseline model, time discounting is the only cost of delaying deal execution to conduct additional due diligence. In practice, there are other costs associated with delay. We extend the model to incorporate such costs here and explore their quantitative relevance in Section V.

B.1. Exogenous Risk of Deal Failure

A merger agreement may fail for reasons unrelated to due diligence. For example, the government may fail to approve the merger agreement due to anti-trust concerns, or a change in the economic environment may eliminate the potential gains from trade.

Extending the baseline model to account for an exogenous risk of deal failure is straightforward. Let λ denote the (exogenous) deal failure arrival rate and assume that if it arrives prior to deal completion then the seller gets his reservation value and the acquirer gets zero. Therefore, all players behave as if the common discount rate were $r + \lambda$, and equilibrium prices, strategies and comparative statics depend only on this sum. The only results from the baseline model that depend on the relative magnitudes of r and λ are those pertaining to the expected time to completion and the probability of deal completion, both of which are decreasing in λ, holding $r + \lambda$ fixed (see the Internet Appendix).

B.2. Flow Costs

In the baseline model, due diligence does not consume any of the acquirer's resources. In practice, there are costs to performing due diligence. For example, the acquirer typically needs to hire accountants to verify financial statements and/or inspectors to evaluate the condition of physical capital. When due diligence is costly, the acquirer may wish to terminate the transaction if enough negative information is revealed. In this section, we extend the baseline model to allow for costly due diligence and deal termination.

Specifically, assume that the acquirer pays a flow cost c until she completes the deal, which happens at τ, or terminates the deal, which happens at ζ. The acquirer chooses a completion and termination time to maximize her payoff

$$F_B^c(q|P) = \sup_{\tau,\zeta} \mathbb{E}_q \left[\mathbb{I}_{\{\tau<\zeta\}} e^{-r\tau}(V(q_\tau) - P) - \int_0^{\min\{\tau,\zeta\}} e^{-rt} c dt \right].$$

Fixing any $P \in (k, V_H)$, the acquirer's optimal strategy is to complete the deal for beliefs above an upper threshold $\hat{b}(P)$, to terminate the deal for beliefs below a lower threshold $\hat{a}(P)$, and to conduct due diligence for beliefs in between.[21] Results from the baseline model can be extended to the case with costly due diligence. For any $P > k$, the acquirer's execution threshold remains above the socially efficient execution threshold. In addition, the termination threshold is also above the socially efficient termination threshold. Intuitively, because the acquirer does not capture all of the surplus from a completed deal, she "gives up" prior to when doing so is socially optimal. In that sense, there is both inefficient execution and inefficient termination.

The profit maximizing and socially efficient outcome can be obtained with an unconditional transfer and a contingent price $P = k$ (i.e., Proposition 11 still holds). In fact, absent discounting ($r = 0$) there is an equivalence between the use of *break-up fees* (i.e., a transfer from the acquirer to the seller for failing to complete the transaction) and the use of unconditional transfers. Precisely, if $r = 0$ then a contract $C = (U, P)$ is functionally equivalent to a contract with a break-up fee of $B = U$ and a contingent price $P + U$, and the optimal mechanism can be implemented with a price/break-up fee pair. However, this result is special to the no discounting case.

PROPOSITION 12 (Break-Up Fees): *With costly due diligence:*

(i) Without discounting ($r = 0$), the seller optimal mechanism is socially optimal and can be implemented with a break-up fee $B = F_B^c(q|k)$ and a contingent price $P = k + B$;

(ii) With discounting ($r > 0$), if the seller optimal break-up fee and contingent price

[21] The upper and lower thresholds can be determined by solving a system consisting of an ODE for the acquirer's value function and four boundary conditions (value matching and smooth pasting at each boundary) and four unknowns (two constants in the acquirer's value function and the two boundaries).

pair leads to due diligence (i.e., $\hat{a} < q_0 < \hat{b}$), then the seller optimal pair is not socially optimal.

It is apparent from (ii) that, with discounting, a break-up fee is not functionally equivalent to an unconditional transfer. The combination of discounting and a break-up fee distorts the acquirers incentives for deal execution and termination, while an unconditional transfer does not. In order to prove (ii), we show that the unique contract that achieves the social optimum is $(B, P) = (0, k)$, which is clearly not seller optimal. The non-trivial part of the proof is that no other (B, P) pair can achieve socially optimal due diligence.

V. Quantitative Implications

In this section, we explore the quantitative implications of our model within the context of public M&A. We begin by estimating parameters of the baseline model using empirical moments from the literature. The baseline model is able to match the moments in the data. However, the baseline model also requires an unreasonably high discount rate to do so. This finding suggests that the discount rate is proxying for other costs associated with delaying execution, and the baseline model likely overstates the magnitude of the distortion. To understand the quantitative importance of these other types of delay costs, we estimate the model variations with exogenous failure intensity (λ) and with flow costs (c) introduced in Section IV.B. Both models perform well with a 10% discount rate target.

We then perform two counterfactual exercises. First, we compare the equilibrium surplus in each of the three models to the social optimum. Second, we explore the extent to which break-up fees can mitigate the distortion in the flow cost model with time discounting.

A. Moments and Estimation Procedure

After we normalize $V_H = 1$ and $V_L = 0$, the baseline model has four remaining parameters: r, ϕ, k, and q_0. We use the following four model moments to estimate these parameters:

(i) The offer premium, $(P^* - k)/k$,

(ii) The probability of deal failure, $1 - \mathbb{P}_0(\tau < \infty)$,

(iii) The expected conditional time to completion, $\mathbb{E}_0[\tau | \tau < \infty]$,

(iv) The ratio of price to maximum willingness to pay, $P^*/\mathbb{E}_0[V_\theta | q_0 = b(P^*)]$.

We obtain the data counterparts for these moments from the literature. Betton et al. (2014) find that the average offer premium is 45% in their sample of over 6,000 take-over bids for US public targets in 1980-2008. We set the failure probability to 0.125 based on Heath and Mitchell (2022). Offenberg and Pirinsky (2015) find that it takes on average 135 days to complete a merger after publicly announcing it. Therefore, we set the time to completion equal to 135/365. Based on Gorbenko and Malenko (2014), we target a price relative to maximum willingness to pay of 88.1%.[22] Table I summarizes the targeted moments.

Moment	Model	Data	Data Source
Offer premium	$\frac{P^*-k}{k}$	0.45	Betton et al. (2014)
Failure probability	$1 - \mathbb{P}_0(\tau < \infty)$	0.12	Heath and Mitchell (2022)
Time to completion	$\mathbb{E}_0[\tau \| \tau < \infty]$	0.37	Offenberg and Pirinsky (2015)
Price versus value	$\frac{P^*}{\mathbb{E}_0[V_\theta \| q_0 = b(P^*)]}$	0.88	Gorbenko and Malenko (2014)

Table I. Model and data moments.

The failure intensity and flow cost models have a fifth parameter (λ and c, respectively) and thus require an additional moment. For these two models, we target a 10% discount rate.

[22]Gorbenko and Malenko (2014) find that in takeovers strategic acquirers pay on average 85.1% of their willingness to pay while financial acquirers pay on average 92.7% of their willingness to pay. In their data set, 211 of the 349 auctions are won by strategic acquirers. Our target moment is the weighted average $(211/349) \times 85.1\% + (138/349) \times 92.7\% = 88.1\%$.

Similar to Hennessy and Whited (2005) and Taylor (2010), we obtain the model parameters $\mathcal{P}$ by minimizing the squared distance between the vector of model moments $M(\mathcal{P}) \in \mathbb{R}^n$, which are a function of the model parameters, and the vector of data moments $D \in \mathbb{R}^n$:

$$\min_{\mathcal{P}} \left(M(\mathcal{P}) - D\right)' I \left(M(\mathcal{P}) - D\right),$$

where we use the identity matrix $I \in \mathbb{R}^{n \times n}$ as the weight matrix. We adopt standard techniques to estimate the parameters of each model.[23]

B. *Estimation Results*

By suitable choice of the four parameters, r, ϕ, k, q_0, the baseline model can exactly match the targeted moments. However, the implied discount rate of 234% is very large. The failure-intensity and flow-cost models each have an extra parameter, and in these models we can exactly match the targeted moments while also setting the discount rate to a more reasonable value, such as 10%. Table II contains the estimated parameters from each of the three models.

Parameter	Baseline	Failure Intensity	Flow Cost
Learning Speed ϕ	2.47	0.93	0.51
Reservation value k	0.57	0.57	0.47
Prior q_0	0.81	0.91	0.75
Failure rate λ	–	0.23	–
Flow cost c	–	–	0.02
Interest rate r	2.34	0.10	0.10

Table II. Parameter estimates from each of the three models.

Both the prior and reservation value are reasonably stable across the three models. The reservation values correspond to 47-57% of the bidders' valuation for a high-type asset.

[23]Specifically, to estimate the parameters for the baseline and exogenous failure model, we use 1000 random draws to initialize the parameters. For each draw, we perform a local optimization routine. We then select the parameters that (globally) minimize the objective function. For the flow cost model, we use 250 random draws to initialize parameters. For each draw, we run a simulated annealing algorithm (Kirkpatrick et al., 1983) and then select the parameters that minimize the objective function.

The estimated failure intensity corresponds to a $20.5\% = 1 - e^{-0.23}$ chance of deal failure for exogenous reasons over a one year period. The estimated flow cost of performing due diligence corresponds to 2% of the acquirer's value for a high-type asset on an annualized basis.

C. Counterfactuals

To further explore the quantitative implications, we first quantify the surplus lost in equilibrium due to inefficient due diligence compared to the social optimum. The results are summarized in Table III.

Surplus	Baseline	Failure Intensity	Flow Cost
Equilibrium surplus	0.18	0.31	0.25
Social optimum surplus	0.25	0.34	0.28
Surplus loss in equilibrium	0.28	0.10	0.11
Surplus loss/price	0.08	0.04	0.04
Socially optimal upfront/price	0.44	0.61	0.58

Table III. Surplus and inefficiency for the three models.

The third row of Table III implies that, in the baseline model, equilibrium surplus is 28% lower than the social optimum. This loss in surplus corresponds to 8% of the bid price. The loss in surplus is lower in the failure intensity and flow cost models (10% and 11%, respectively), but remains economically significant and corresponds to 4% of the bid price in both models.

Recall from Proposition 11 that if we enrich the contract space to allow for unconditional transfers then the seller optimal mechanism achieves the social optimum. However, in order to do so, the unconditional transfer must be strikingly large: it corresponds to 44-61% of the optimal price as indicated by the last row of Table III. The required magnitude of the unconditional transfer combined with the unappealing features of the seller optimal mechanism (discussed in Section IV.A) provide a potential explanation for their lack of use in public M&A.

Our second counterfactual exercise is to explore the quantitative extent to which break-up fees can be used to mitigate the economic distortion in the flow cost model.

Chen et al. (2022) document that 21% of M&A deals have a bidder break-up provision with an average break-up fee size of 3.6% of the deal value. Bates and Lemmon (2003) find similar results and argue that break-up fees enable the seller to lock in the surplus of the deal (whether or not it is executed).

Recall from Proposition 12, that absent time discounting, break-up fees enable the seller to simultaneously implement the social optimum and extract the full surplus. However, the same is not true with time discounting. Therefore, it remains a quantitative question as to the extent to which break-up fees are an effective way to increase surplus or simply a tool for rent extraction. Our findings suggest it is primarily the latter.

For this exercise, we use the parameter estimates for the flow cost model in Table II. For each level of break-up fee, we compute the seller optimal price and plot it in Figure 9A. Starting from no break-up fee, the optimal price initially increases with the break-up fee until the acquirer's value reaches zero (as illustrated in Figure 9C), at which point the price decreases with the break-up fee in order to satisfy the acquirer's participation constraint.

The seller's payoff for each contract is plotted in Figure 9B. The seller optimal break-up fee (denoted B^*) is 0.23, which corresponds to roughly 30% of the price. Notice that the seller's payoff is significantly higher with a break-up fee of B^* than without a break-up fee. The total surplus also increases (Figure 9D) from 89% to 96% of the social optimum. However, most of the seller's gain materializes by extracting rent from the acquirer rather than more efficient of due diligence. Comparing the optimal contract with break-up fees to the optimal contract without break-up fees, the seller's payoff increases from 0.18 to 0.26, and the acquirers payoff falls from 0.07 to 0. Thus, while the seller's payoff increases by 0.08 (or 44%), only 0.01/0.08=12.5% of the increase derives from more efficient due diligence.

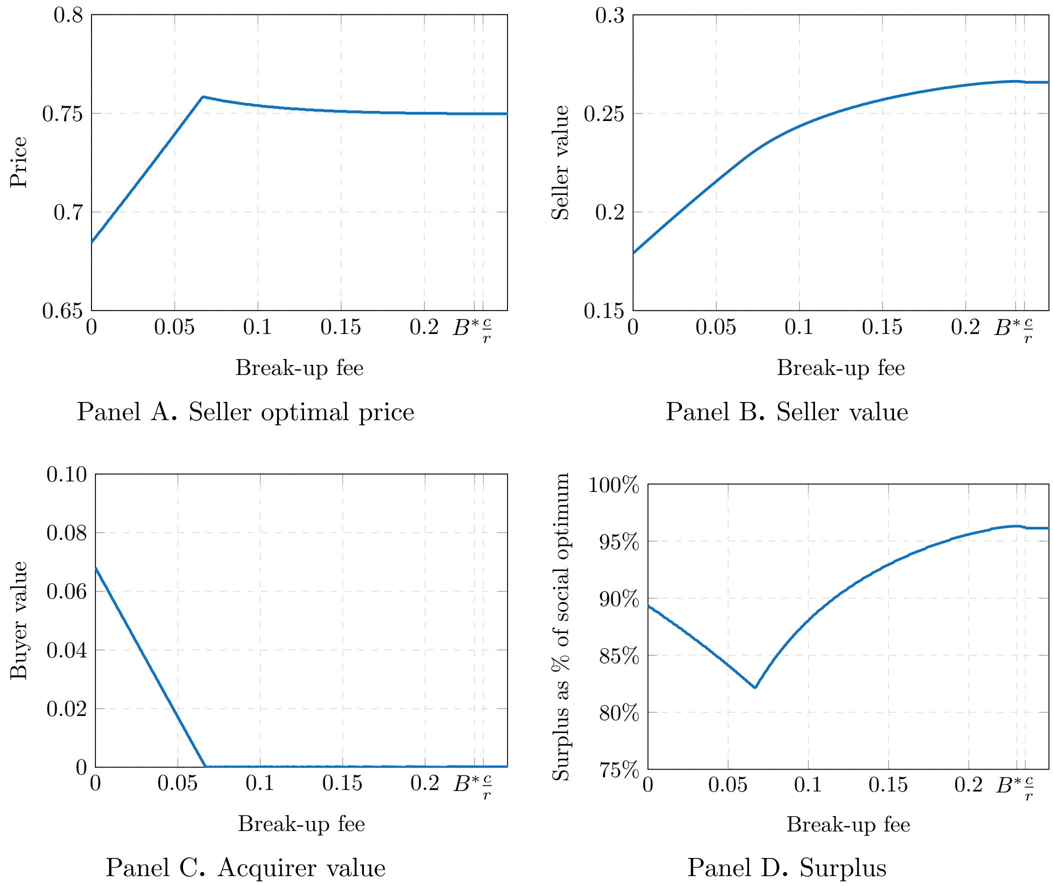


Figure 9. Effect of break-up fee on price and social surplus.

VI. Conclusion

Due diligence is common practice prior to the execution of large transactions. We propose a model of the due diligence process and analyze its effect on prices, payoffs, and deal completion. If the acquirer has the right to conduct due diligence and determine if/when to execute, then the asset she acquires when the seller accepts the offer is a real option, which has important economic implications for trading outcomes.

In equilibrium, the acquirer engages in "too much" due diligence relative to the social optimum and can extract a positive surplus even with perfect competition among bidders. Our quantitative results suggest the magnitude of the distortion is economically significant. Nevertheless, allowing for due diligence can improve both total surplus and the seller's payoff compared to a setting with no due diligence. We explore how the acquirer's speed of conducting due diligence affects the equilibrium price, the likelihood of deal completion and the duration of the due diligence phase. We use our framework to explore the timing of due diligence, bidder heterogeneity, and contractual arrangements such as break-up fees.

Appendix

In the appendix, we will sometimes use beliefs transformed into a log-likelihood ratio, denoted $z \equiv \ln\left(\frac{q}{1-q}\right)$. The dynamics of the log-likelihood of the belief process, $Z_t = \ln\left(\frac{q_t}{1-q_t}\right)$, are given by

$$dZ_t = \frac{\phi^2}{2}\left(2p(Z_t) - 1\right)dt + \phi dB_t,$$

where $p(z) \equiv \frac{e^z}{1+e^z}$, which transforms a likelihood ratio to a probability. We use $\beta(P)$ to denote the log-likelihood ratio of $b(P)$.

Let $\mathcal{A}$ denote the infinitesimal generator of Z, which is defined to act on suitable functions f by $\mathcal{A}f(z) = \lim_{t\downarrow 0} \frac{\mathbb{E}_z[f(Z_t)]-f(z)}{t} = \frac{\phi^2}{2}\left((2p(z)-1)f' + f''\right).$

A. *Proofs for Section I*

Proof of Lemma 1. We will construct the value function $F_B(z|P)$ given the candidate strategy $\tau^*(P) = \inf\{t > 0 | q_t \geq b(P)\}$ and show that it solves the Hamilton-Jacobi-Bellman equation

$$0 = \max\left\{-rF_B(z|P) + \mathcal{A}F_B(z|P), (\mathbb{E}_z[V_\theta] - P) - F_B(z|P)\right\},$$

and is smooth (C^1, C^2 a.e.), which is sufficient to ensure that $\tau^*(P)$ solves the acquirer's problem (Harrison, 2013, Theorem 5.1).

Given the candidate $\tau^*(P)$, $F_B(z|P) = \mathbb{E}_z[V_\theta] - P$ for $z \geq \beta(P) \equiv \ln\left(\frac{b(P)}{1-b(P)}\right)$, and F_B satisfies $rF_B(z|P) = \mathcal{A}F_B(z|P)$ for $z < \beta(P)$. Thus, there are two things to check in order to verify F_B satisfies the HJB: (1) that $-rF_B(z|P) + \mathcal{A}F_B(z|P) \leq 0$ for $z > \beta(P)$ and (2) that $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$ for $z < \beta$.

For (1), given $F_B(z|P) = \mathbb{E}_z[V_\theta] - P$, we can compute

$$-rF_B(z|P) + \mathcal{A}F_B(z|P) = \frac{r(P + e^z(P - V_H) - V_L)}{1+e^z}.$$

The RHS has the same sign as $P + e^z(P - V_H) - V_L$, which is decreasing in z (since

$P < V_H$) and at $\beta(P)$ is equal to

$$P + e^{\beta(P)}(P - V_H) - V_L = \frac{-P + V_L}{u - 1} < 0,$$

which verifies (1). For (2), for $z < \beta(P)$, $F_B(z|P)$ solves $rF_B(z|P) = \mathcal{A}F_B(z|P)$. It is therefore of the form

$$C_1 \frac{e^{uz}}{1 + e^z} + C_2 \frac{e^{\hat{u}z}}{1 + e^z}$$

with $(u, \hat{u}) = \frac{1 \pm \sqrt{1 + \frac{8r}{\phi^2}}}{2}$. Furthermore, F_B is bounded and satisfies the boundary condition

$$F_B(\beta(P)|P) = \mathbb{E}_z[V_\theta] - P$$

which imply that

$$\begin{aligned} C_1 &= e^{-\beta(P)u} \left(- \left(1 + e^{\beta(P)}\right) P + e^{\beta(P)} V_H + V_L \right), \\ C_2 &= 0. \end{aligned}$$

To see that $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$, define

$$G(z) = \frac{\partial \left(F_B(z|P) - (\mathbb{E}_z[V_\theta] - P) \right)}{\partial z}.$$

By construction, $G(\beta(P)) = 0$. Furthermore, $G(z)$ has the same sign as

$$\frac{(1 + e^z)^2}{e^z} G(z)$$

Differentiating this function with respect to z gives

$$\frac{\partial \left(\frac{(1+e^z)^2}{e^z} G(z) \right)}{\partial z} = u \left(e^z + 1\right) (P - V_L) e^{(u-1)z} \left(\frac{u(V_L - P)}{(u - 1)(P - V_H)} \right)^{-u} > 0,$$

which shows that $G(z) < 0$, and therefore $F_B(z|P) \geq \mathbb{E}_z[V_\theta] - P$, for $z < \beta(P)$.

Finally, $F_B(z|P)$ is C^2 everywhere except at $z = \beta(P)$ by construction and it is

C^1 at $z = \beta(P)$ since $\beta(P)$ follows from the smooth pasting condition $F_B'(\beta(P)^-|P) = F_B'(\beta(P)^+|P)$. Therefore, we can apply Theorem 5.1 of Harrison (2013), which proves that $\tau^*(P)$ is the optimal stopping time. $\square$

REMARK 2: *In the proof of Lemma 1, we allow V_L to be arbitrary because this proof will also be used when we study the model with common knowledge of gains from trade (Section II.A) where $V_L > k$. Henceforth, we normalize $V_L = 0$ without loss when doing so will not cause confusion.*

Consider the stopping problem (HSP). Let $h(t, q_t) = e^{-rt}(P_0(q_t) - k)$ from stopping at time t. By Dynkin's formula, the payoff under an arbitrary stopping time with finite expectation is

$$\mathbb{E}_q[h(\tau, q_\tau)] = h(0, q) + \mathbb{E}_q\left[\int_0^\tau \hat{\mathcal{A}}h(s, q_s)ds\right]$$

where $\hat{\mathcal{A}}$ is the characteristic operator of the process (t, q_t) under $\mathcal{P}$. That is,

$$\hat{\mathcal{A}}h(q,t) = re^{-rt}\left(\frac{\gamma}{2}q^2(1-q)^2P_0''(q) - (P_0(q) - k)\right). \tag{A.1}$$

LEMMA A.1: *There exists a $\bar{k} \in (0, V_H)$ such that $\hat{\mathcal{A}}h$ satisfies single crossing with respect to q for all $k \geq \bar{k}$.*

Proof. The expression inside the parentheses of (A.1) is independent of t. Thus, it suffices to show that it this expression satisfies single crossing w.r.t. q for all $k \geq \bar{k}$. The first term of the expression,

$$\frac{\gamma}{2}q(1-q)P_0''(q) = -\frac{2(q-1)^2q^2V_H}{(q-u)^3},$$

is weakly positive and equal to zero at $q = 1$. Furthermore, $P_0(q)$ is strictly increasing in q. Hence, the expression attains its minimum at $q = 1$, which is $-(V_H - k) < 0$. Further, the expression is increasing in k and equal to k at $q = 0$. To characterize $\bar{k}$, note that

$$\frac{\gamma}{2}q(1-q)P_0''(q) - P_0(q)$$

is a rational function in q and therefore has a finite number of local minima, denoted $\{q_1, q_2, ..., q_n\}$, on $(0,1)$. Let $k_i = P_0(q_i) - \frac{\gamma}{2}q_i(1-q_i)P_0''(q_i) < V_H$ and define

$$\bar{k} = \max\{0, k_1, k_2, ..., k_n\}.$$

For all $k \geq \bar{k}$, all local minima of the expression are weakly positive, while the global minimum (attained at $q = 1$) is negative. Hence, for all such k, the expression crosses zero exactly once and from above, as desired. □

LEMMA A.2: *There exists a $\bar{\gamma}$ such that $\hat{A}h$ satisfies single crossing with respect to q for all $\gamma > \bar{\gamma}$.*

Proof. The constant $u > 1$ is decreasing in γ since $u = \frac{1+\sqrt{\frac{8}{\gamma}+1}}{2} > 1$. Define

$$g(q) = -(q-u)^3 \frac{e^{rt}}{r}\hat{A}h,$$

which has the same sign has $\hat{A}h$, since $u > 1$ for any $\gamma \in [0, \infty)$. Hence, it suffices to demonstrate that g satisfies single crossing for $\gamma \geq \bar{\gamma}$.

Observe that $g(0) = u^3 k > 0$, $g(1) = (u-1)^3(k - V_H) < 0$, and $g'(0) = 3ku^2 - (u-1)u^2 V_H < 0$. Furthermore, $g''(q)$ is quadratic in q and $g''(0) = 6ku + 4(1+(-1+u)u)V_H > 0$. Finally, $g''(1) = 6k(-1+u) + 2(5+u(-5+2u))V_H$, which is positive for u sufficiently close to 1. Define $\bar{\gamma}_1 = \inf\{\gamma | g''(1) > 0\}$, and assume from now on that $\gamma > \bar{\gamma}_1$, which implies that $g''(q)$ changes sign either never or twice on $q \in [0,1]$. We break the remainder of the proof into two cases:

Case 1: $k/V_H > 1/4$. For u sufficiently close to one, using the fact that $\frac{k}{V_H} > \frac{1}{4}$, we get that

$$g'''(q) = -6(k + (3 - 8q + u)V_H) < 0.$$

Define $\bar{\gamma}_2 = \inf\{\gamma | g'''(q) < 0 \ \forall q \in [0,1]\}$, and assume from now on that $\gamma > \max_{i \in \{1,2\}} \bar{\gamma}_i$. As a result $g'' > 0$, which implies that g' crosses zero at most once (it is first negative and then positive or always negative) and as a result g crosses zero once from above, as desired.

Case 2: $k/V_H \leq 1/4$ We already established that $g''(0) > 0$ and $g''(1) > 0$. For $u = 1$, the largest solution to $g''(q) = 0$ is given by

$$\frac{3k + 12V_H + \sqrt{9k^2 - 72kV_H + 48V_H^2}}{24V_H} < \frac{3 + 12 + \sqrt{48}}{24} < 1.$$

This implies that for u sufficiently close to one, there exists a $\hat{q} < 1$ such that $g''(q) > 0$ for $q > \hat{q}$. Define

$$\bar{\gamma}_3 = \inf\left\{\gamma \middle| \exists\ q < \frac{3 + 12 + \sqrt{48}}{24}\ \textit{such that}\ g''(q) = 0\right\}.$$

and assume from now on that $\gamma > \max_{i\in\{1,2,3\}} \bar{\gamma}_i$.

Therefore, for $q > \hat{q}$ we have that $g'(q)$ crosses zero at most once and if it does then it crosses from below. This implies that $g(q)$ crosses zero at most once for $q > \hat{q}$.

For $q \in [0, \hat{q})$ and for u sufficiently close to one the operator is positive because

$$\lim_{u\to 1} \hat{\mathcal{A}}h(q,t) = e^{-rt}r\left(k + \frac{2q^2V_H}{1-q}\right) > 0.$$

Define $\bar{\gamma}_4 = \inf\left\{\gamma \middle| g(q) > 0\ \forall q \in \left[0, \frac{3+12+\sqrt{48}}{24}\right]\right\}$ and let $\bar{\gamma} = \max_{i\in\{1,2,3,4\}} \bar{\gamma}_i$. The last two steps imply that for $\gamma \geq \bar{\gamma}$, $\hat{\mathcal{A}}h$ satisfies single crossing. □

Proof of Lemma 2. Lemma A.1 and A.2 imply that $\hat{\mathcal{A}}h$ satisfies single crossing and therefore the optimal stopping strategy for the accompanying stopping problem is a threshold strategy, see (Dixit et al., 1994, Ch. 5). □

Proof of Lemma 3. Clearly, (HSP) provides an upper bound on the seller's equilibrium payoff. We claim that $P_S(q_0)$ (uniquely) achieves this bound. To verify the claim, note that $b(P^*) = b(P_0(q^*)) = q^*$. For $q_0 > q^*$, the claim is immediate since both involve immediate execution at $P_0(q_0)$. For $q_0 \leq q^*$, the acquirer's stopping rule in the due diligence game is τ_{hyp} and the seller's payoff is $E_q[e^{-r\tau_{hyp}}(P_0(q^*) - k)]$, which is exactly the same as the solution to (HSP) given Assumption 1. Moreover, any price different other than $P_S(q_0)$ will lead to a strictly different (and thus lower) payoff for the seller than obtained in the solution to (HSP). $\square$

Proof of Proposition 1. From Lemma 1 it follows that the threshold stopping rule $b(P)$ is aquirer-optimal for any price P. Given this stopping rule, Lemma 3 shows that the seller's payoff is uniquely maximized by the price offer

$$P_S(z) = \begin{cases} P^* & \text{if } q \leq b(P^*) \\ P_0(q) & \text{if } q > b(P^*). \end{cases}$$

By standard arguments, due to competition among bidders, the winning offer must maximize the seller's payoff. $\square$

Proof of Lemma 4. The statement in (i) follows from the fact that q_t is a bounded martingale with continuous sample paths. Therefore, either q_t reaches $b(P^*)$ and the deal is completed, or q_t converges to zero and the deal is never completed. Since $\mathbb{E}_{q_0}[q_{\tau^*}] = q_0 = \mathbb{P}(\tau^* < \infty)b(P^*)$, we have $\mathbb{P}(\tau^* < \infty) = \frac{q_0}{b(P^*)}$.

For (ii), we need to calculate $\mathbb{E}_z[\tau^*|\tau^* < \infty]$. Observe that conditional on θ, Z_t^θ is a Brownian motion with drift $\pm\frac{\phi^2}{2}$ and volatility ϕ. Using Harrison (2013, p. 14), we can then explicitly calculate

$$\mathbb{P}_z(\tau^* \leq t|\theta = H) = 1 - N\left(\frac{(\beta(P^*) - z) - \frac{\phi}{2}t}{\phi\sqrt{t}}\right) + e^{\beta(P^*)-z}N\left(\frac{-(\beta(P^*) - z) - \frac{\phi}{2}t}{\phi\sqrt{t}}\right),$$
$$\mathbb{P}_z(\tau^* \leq t|\theta = L) = 1 - N\left(\frac{(\beta(P^*) - z) + \frac{\phi}{2}t}{\phi\sqrt{t}}\right) + e^{-(\beta(P^*)-z)}N\left(\frac{-(\beta(P^*) - z) + \frac{\phi}{2}t}{\phi\sqrt{t}}\right),$$

where $N(\cdot)$ is the cumulative distribution function of a standard normal random variable. Using these conditional probabilities, we can obtain the cumulative distribution function of τ^* conditional on completion

$$\mathbb{P}_z(\tau^* \leq t|\tau^* < \infty) = \frac{p(z)\mathbb{P}_z(\tau^* \leq t|\theta = H) + (1 - p(z))\mathbb{P}_z(\tau^* \leq t|\theta = L)}{\frac{p(z)}{p(\beta(P^*))}}.$$

Differentiating this cumulative distribution function with respect to t yields the probability distribution function

$$\mathbb{P}_z\left(\tau^* = t | \tau^* < \infty\right) = \frac{(\beta(P^*) - z)e^{-\frac{\left(t\phi^2 - 2(\beta(P^*) - z)\right)^2}{8t\phi^2}}}{\sqrt{2\pi}t^{3/2}\phi},$$

which allows us to calculate

$$\mathbb{E}_z\left[\tau^* | \tau^* < \infty\right] = \int_0^\infty t\mathbb{P}_z\left(\tau^* = t | \tau^* < \infty\right) dt = \frac{2}{\phi^2}(\beta(P^*) - z).$$

□

Proof of Proposition 2. Because $P^*(\phi)$ uniquely maximizes $F_S(q_0|P, \phi)$, which is continuous in P and ϕ, Berge's Theorem (Kreps, 2013, Proposition A4.7 and Corollary A4.8) implies that P^* is continuous in ϕ. We turn now to establishing P^* is monotone in ϕ. Given a price P, the seller's payoff for $z < \beta(P)$ is given by

$$F_S(z|P) = \frac{(P-k)e^{uz}\left(\frac{Pu}{(u-1)(V_H - P)}\right)^{-u}\left(\frac{Pu}{(u-1)(V_H-P)} + 1\right)}{e^z + 1}.$$

To maximize the seller's payoff with respect to P, first derive

$$\frac{\partial F_S(z|P)}{\partial P} = -\frac{e^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}\left((u-1)V_H^2(P(u-1) - ku) + P^2(u-2)V_H + P^3\right)}{P(u-1)\left(e^z+1\right)\left(P - V_H\right)^2},$$

and note that because $P \in (k, V_H)$, $\partial F_S(z|P)/\partial P$ has the same sign as

$$w(P) \equiv -\left((u-1)V_H^2(P(u-1) - ku) + P^2(u-2)V_H + P^3\right).$$

Now suppose P^* were not monotone in ϕ. Then there would exist some price $\hat{P}^* \in (k, V_H)$ such that $w(\hat{P}^*|u_1) = w(\hat{P}^*|u_2) = 0$ for distinct $u_1, u_2 > 1$. However, $w(P|u)$ is quadratic in u with roots,

$$\frac{k + P^2 - 2P + \frac{\sqrt{k^2V+2kP^2(2P-3V)+P^3(P(V-4)+4V)}}{\sqrt{V}}}{2(k-P)} < 1,$$
$$\frac{k + P^2 - 2P - \frac{\sqrt{k^2V+2kP^2(2P-3V)+P^3(P(V-4)+4V)}}{\sqrt{V}}}{2(k-P)} > 1,$$

producing a contradiction. Therefore, the price P^* must be monotone in ϕ. Finally, as $\phi \to \infty$, we have $u \to 1$ and $w(P) \to (P^2V_H - P^3)$. Therefore, $P^* \to V_H$ as $\phi \to \infty$. Hence, P^* is montonically *increasing* in ϕ.

Given that increasing ϕ increases both P^* and $b(P)$ for any P (immediate from the closed-form in (2)), we have that $b(P^*)$ is increasing in ϕ. Also, because $b(P)$ is increasing in ϕ, we have that $P_0(q)$ is decreasing in ϕ. Statement (i) then follows from the fact that $P_S(q_0) = \max\{P^*, P_0(q_0)\}$. Next, the probability of deal completion is $\min\{1, \frac{q_0}{b(P^*)}\}$. Statement (ii) then follows then follows from the fact that $b(P^*)$ is increasing in ϕ.

For statement (iii), using Lemma 4(ii), for $q_0 \in [b(P^*|\phi_1), b(P^*|\phi_2))$ the expected completion time increases while for $q_0 \geq b(P^*|\phi_2)$ it remains unchanged. For $q_0 < b(P^*|\phi_1)$, we have that

$$\begin{aligned}
&\mathbb{E}\left[\tau^*|\tau^* < \infty, \phi_2\right] - \mathbb{E}\left[\tau^*|\tau^* < \infty, \phi_1\right] \\
&= \left(\frac{2}{\phi_2^2}\ln\left(\frac{b(P^*|\phi_2)}{1-b(P^*|\phi_2)}\right) - \frac{2}{\phi_2^1}\ln\left(\frac{b(P^*|\phi_1)}{1-b(P^*|\phi_1)}\right)\right) - \underbrace{\left(\frac{2}{\phi_2^2} - \frac{2}{\phi_1^2}\right)}_{<0}\ln\left(\frac{q_0}{1-q_0}\right),
\end{aligned}$$

which is increasing in q_0 and negative for q_0 sufficiently small. $\square$

Proof of Proposition 4. Part (iii) of the proposition is immediate since both cases involve immediate execution. We demonstrate (i) and (ii) in three steps. Let $z^* \equiv \ln\left(\frac{q^*}{1-q^*}\right)$. Define $\underline{z} < b(0)$ as the solution to $\mathbb{E}_{\underline{z}}\left[V_\theta - k\right] = 0$.

Step 1: There is exists a z_1 such that social surplus is higher with due diligence for $z < z_1$. For $z \leq \underline{z}$, social surplus is strictly positive with due diligence and equal to zero without due diligence:

$$F_S(z|P_S(z)) + F_B(z|P_S(z)) = \mathbb{E}_z\left[e^{-r\tau(P^*)}(V(q^*) - k)\right] > 0 = \max\left\{\mathbb{E}_z\left[V_\theta - k\right], 0\right\}.$$

Both with and without due diligence, surplus is continuous in z. Hence, the inequality continues to hold in a neighborhood above $\underline{z}$. Let z_1 be the largest threshold for which the inequality holds for all $z < z_1$.

Step 2: There is exists a z_2 such that social surplus is lower with due diligence for $z \in (z_2, z^*)$. From Proposition 3, the socially optimal execution threshold, denoted $\beta(k)$, is below z^*. Therefore, maximal social surplus is achieved without due diligence for $z \in (\beta(k), z^*)$, while the outcome with due diligence is not socially optimal. By continuity, social surplus without due diligence remains higher in a neighborhood below $\beta(k)$. Let z_2 be the smallest belief for which social surplus is weakly higher without due diligence.

Step 3: $z_1 = z_2$. For $z \in (\underline{z}, z^*)$, we can differentiate the difference in social surplus with respect to z

$$\frac{\partial}{\partial z}\left(\mathbb{E}_z\left[V_\theta - k\right] - \left(F_S(z|P^*) + F_B(z|P^*)\right)\right).$$

This derivative has the same sign as

$$\begin{aligned}
&\left(\frac{(1+e^z)^2}{e^z}\right)\frac{\partial}{\partial z}\left(\mathbb{E}_z\left[V_\theta - k\right] - F_S(z|P^*) - F_B(z|P^*)\right) \\
=&\frac{e^{(u-1)z}\left((u-1)e^z + u\right)\left(\frac{P^*u}{(u-1)(V_H - P^*)}\right)^{-u}\left(P^*uV_H - k(P^* + (u-1)V_H)\right)}{(u-1)(P^* - V_H)} + V_H,
\end{aligned}$$

which is monotone in z and therefore crosses zero at most once. Since the derivative of the difference changes sign at most once on $(\underline{z}, z^*)$, the difference in social surplus crosses zero at most once on $(\underline{z}, z^*)$. Hence, $z_1 = z_2$. Claims (i) and (ii) of the Proposition have thus been established, where $\bar{q} = p(z_1) = p(z_2)$. □

Proof of Proposition 3. The proof of the first statement is analogous to the proof of Lemma 1 after replacing P with k. That $k < P^*$ follow from the fact that if $P = k$

then the seller's payoff is zero whereas for any $P > k$, the seller's payoff is strictly positive. Hence, $P^* > k$. The second statement of the proposition then follows from the fact that $b(P^*) > b(k)$. Hence, by the same argument as used in the proof of Lemma 4(i), the probability of deal completion in the socially optimal policy is $q_0/b(k) > q_0/b(P^*)$. $\square$

B. Proofs for Section II

Definition of Equilibrium with Dynamic Bidding: A quadruple $\{P, \nu, \tau, q\}$ such that

1. *Seller optimality:* $\nu(t) \in \arg\sup_{T\geq t} \mathbb{E}_t[e^{-r(T+\tau(P_T,T)-t)}(P_T - k)]$. Let $G_t(\omega)$ denote the seller's payoff under the solution.

2. *Acquirer optimality:* given any offer P accepted at any date t, $\tau(P,t)$ solves the acquirer problem $\sup_{\tau\geq t} \mathbb{E}_t[e^{-r(\tau-t)}(V(q_\tau) - P)]$.

3. *Buyer competition:* Given the seller's continuation payoff at any $t \leq \nu$, $G_t(\omega)$, there does not exist an offer y such that $G_t(\omega) < \mathbb{E}_t[e^{-r(\tau(y,t)-t)}(y - k)]$.

Proof of Proposition 5. Recall that the seller's payoff in the baseline model is the same as in the solution to (HSP), which we denote by $F_0(q)$. Therefore, it suffices to show that the seller's payoff in any equilibrium with dynamic bidding is equal to $F_0(q)$. Consider the Buyer competition condition and let $y = P_S(q)$, then immediately we have $G_t(\omega) \geq \mathbb{E}_t[e^{-r(\tau(P_S(q_t),t)-t)}(P_S(q_t) - k)] = F_S(q_t|P_S(q_t)) = F_0(q_t)$. Setting $t = 0$, we conclude that the seller's payoff with dynamic bidding is weakly bigger than with static bidding.

Next, we argue $G_t(\omega) \leq F_0(q_t)$. Fix any $\nu \geq t$. Using the tower property, we have

$$\mathbb{E}_t[e^{-r(\nu-t)}e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] = \mathbb{E}_t[e^{-r(\nu-t)}\mathbb{E}_\nu[e^{-r\tau(P_\nu,\nu)}(P_\nu - k)]]$$

And clearly $\mathbb{E}_\nu[e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] \leq \max_P\{\mathbb{E}_\nu[e^{-r\tau(P,\nu)}(P-k)]\}$, therefore

$$\begin{aligned}
\mathbb{E}_t[e^{-r(\nu-t)}e^{-r\tau(P_\nu,\nu)}(P_\nu - k)] &\leq \mathbb{E}_t[e^{-r(\nu-t)}\max_P\{\mathbb{E}_\nu[e^{-r\tau(P,\nu)}(P-k)]\}] \\
&= \mathbb{E}_t[e^{-r(\nu-t)}\mathbb{E}_\nu[F_S(q_\nu|P_S(q_\nu))]] \\
&= \mathbb{E}_t[e^{-r(\nu-t)}F_0(q_\nu)] \\
&\leq F_0(q_t).
\end{aligned}$$

where the first equality is by the definition of P_S, the second equality follows from the previously established fact that $F_S(q|P_S(q)) = F_0(q)$. The last inequality follows because ν acts as a constraint and any constraint on the stopping rule (i.e., $\tau \geq \nu$) in (HSP) is dominated by the solution to the unconstrained problem. Taking the supremum over all ν, we conclude that $G_t(\omega) \leq F_0(q_t)$.

We have therefore established that for any t, $F_S(q_t|P_S(q_t)) \leq G_t(\omega) \leq F_0(q_t) = F_S(q_t|P_S(q_t))$. Letting $t=0$ completes the proof. $\square$

LEMMA A.3: *Suppose that $k < V_L$. Then, there exists a $\underline{\gamma} > 0$ such that for all z_0 the winning offer is $P_0(z_0)$ and the transaction is executed immediately if and only if $\gamma \leq \underline{\gamma}$.*

Proof. First, observe that the seller's payoff for $z \leq b(P)$ can be written as $F_S(z|P) = \psi(P,u)\frac{e^{uz}}{1+e^z}$, where

$$\psi(P,u) = (P-k)\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^{-u}\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}+1\right).$$

Therefore, given any z, picking the seller optimal price (i.e., solving $\max_P F_S(z|P)$) is equivalent to solving $\max_{P\geq P_0(z)}\psi(P,u)$, since the seller either picks the highest price that leads to direct execution $P_0(z)$ or a higher price that induces due diligence.

STEP 1: *Direct execution is optimal for all z_0 if and only if $\psi_P(P,u) \leq 0$ for all $P \in (V_L, V_H)$.*

The "if" part of the claim is immediate. For the other direction, suppose that direct execution is optimal for all z_0 but $\psi_P(\hat{P},u) > 0$ for some $\hat{P} \in (V_L,V_H)$. Take $\hat{z}$ such that $\hat{P} = P_0(\hat{z})$ (such a $\hat{z}$ necessarily exists by continuity of P_0, $P_0(-\infty) = V_L$, and $P_0(\infty) = V_H$). Then for $z_0 = \hat{z}$, the seller can do better by charging a price slightly above $P_0(\hat{z})$, which contradicts direct execution being optimal for all z_0.

STEP 2: *If direct execution is optimal for all z_0 given some fixed $\gamma = \underline{\gamma}$. Then direction execution is optimal for all z_0 for any $\gamma < \underline{\gamma}$.*

We prove this claim by first showing that $\psi_{P,u}(P,u) \leq 0$ for all $P \in (V_L, V_H)$ and $u > 1$. In order to do so, observe that for $P \in (V_L, V_H)$, $\psi_P(P,u)$ has the same sign as $\tilde{\psi}_P(P,u)$, where

$$\begin{aligned}\tilde{\psi}_P(P,u) =& \psi_P(P,u)(u-1)(P-V_H)^2(P-V_L)\left(\frac{u(V_L-P)}{(u-1)(P-V_H)}\right)^u \\ =& k(u-1)u(V_H-V_L)^2 - P^3 + P^2(-uV_H + uV_L + 2V_H + V_L) \\ &- P\left(2\left(-u^2+u+1\right)V_HV_L + u^2V_L^2 + (u-1)^2V_H^2\right) + V_HV_L(-uV_H + uV_L + V_H).\end{aligned}$$

Further, observe that for $P \in (V_L, V_H)$

$$\begin{aligned}\tilde{\psi}_{P,u}(V_L,u) &= (2u-1)(k-V_L)(V_H-V_L)^2 < 0, \\ \tilde{\psi}_{P,u,P}(P,u) &= -2(V_H-V_L)(P+(u-1)V_H - uV_L) < 0,\end{aligned}$$

which establishes that $\tilde{\psi}_{P,u}(P,u)$ is negative at $P = V_L$ and decreasing in P. Hence, it is negative for all for $P \in (V_L, V_H)$. That $\psi_P(P,u)$ has the same sign as $\tilde{\psi}_P(P,u)$ implies that $\psi_{P,u}(P,u) \leq 0$ for all $P \in (V_L, V_H)$ and $u > 1$.

Now, recall that u is decreasing in γ. Therefore, if for some $\underline{\gamma}$ $\psi_P(P, u(\underline{\gamma})) \leq 0$ for all $P \in (V_L, V_H)$ then for any $\gamma \leq \underline{\gamma}$, $\psi_P(P, u(\gamma)) \leq 0$ for all $P \in (V_L, V_H)$. Step 1 then implies that direct execution is optimal for all z_0 and any such $\gamma \leq \underline{\gamma}$.

STEP 3: *There exists a $\underline{\gamma} > 0$ such that direct execution is optimal for all z_0 when $\gamma = \underline{\gamma}$.*

Collecting terms related to u in $\tilde{\psi}_P$, we obtain

$$\begin{aligned}\tilde{\psi}_P(P,u) = & -u(V_H - V_L)\left(k(V_H - V_L) + P^2 - 2PV_H + V_H V_L\right) \\ & + u^2(k-P)(V_H - V_L)^2 - P^3 + 2P^2V_H + P^2V_L - PV_H^2 - 2PV_HV_L + V_H^2V_L.\end{aligned}$$

As $\gamma \to 0$, $u \to \infty$ and $\tilde{\psi}_P(P,u) \to u^2(k-P)(V_H-V_L)^2 < 0$. Therefore, by the continuity of $\tilde{\psi}_P$, there exists a $\underline{\gamma}$ small enough such that $\tilde{\psi}_P(P,u(\underline{\gamma})) < 0$ and direct execution is optimal for all z_0. □

LEMMA A.4: *Suppose that $k < V_L$ and $\gamma > \underline{\gamma}$ then there exist two thresholds $z_a < z_b$ such that*

(a) For $z_0 \in (z_a, z_b)$, the winning offer is $P_0(z_b)$ and the acquirer conducts due diligence until $\tau^*(P_0(z_b))$.

(b) For $z_0 \notin (z_a, z_b)$, the winning offer is $P_0(z_0)$ and the transaction is executed immediately.

Proof. Assume $\gamma > \underline{\gamma}$ then we know delay takes places for some z, see Lemma A.3. Assume delay takes place for $\hat{z}$ and the seller picks a price $\tilde{P} > P_0(\hat{z})$ then define $\tilde{z}$ as the solution to $\tilde{P} = P_0(\tilde{z})$. We then know that delay must take place for $z \in (\hat{z}, \tilde{z})$ since

$$\psi(\tilde{P},u) = \max_{P \geq P_0(\hat{z})} \psi(P,u) \geq \max_{P \geq P_0(z)} \psi(P,u).$$

Assume there is more than one region in which delay takes place. For simplicity, assume there are two (the proof easily extends with more than two). The existence of these two delay regions implies that there exists a $z_1 < z_2 < z_3 < z_4 < z_5$ such that

1. For $z < z_1$, direct execution is optimal because $\lim_{P \to V_L} \psi(P,u) = \infty$ and $\psi(P,u) < \infty$ for any $P \in (V_L, V_H)$ and therefore $\psi_P(P_0(z),u) \leq 0$.

2. For $z \in (z_1, z_2)$, delay takes place and therefore $\psi_P(P_0(z),u) > 0$.

3. For $z \in (z_2, z_3)$, no delay takes place and therefore $\psi_P(P_0(z),u) \leq 0$.

4. For $z \in (z_3, z_4)$, delay takes place and therefore there exists a $z \in (z_3, z_4)$ such that $\psi_P(P_0(z), u) > 0$.

5. For $z > z_5$, no delay takes since $\lim_{P \to V_H} \psi_P(P, u) = (u-1)u(k - V_H)(V_H - V_L)^2 < 0$

This implies that $\psi_P(P, u) = 0$ for at least four values of $P \in (V_L, V_H)$. From the Proof of Lemma A.3, ψ_P has the same sign as $\tilde{\psi}_P$. And $\tilde{\psi}_P$ is a third-order polynomial in P, which can have at most three real roots, which leads to a contradiction. Therefore, there can be at most one region of delay. □

Proof of Proposition 6. The result follows directly from Lemma A.3 and Lemma A.4. □

Proof of Proposition 7. For $\gamma > \underline{\gamma}$, delay is optimal and dynamic bidding strictly dominates static bidding for some beliefs. Therefore, there must exist a $\underline{\underline{\gamma}} \leq \underline{\gamma}$ such that for $\gamma > \underline{\underline{\gamma}}$ dynamic bidding strictly dominates static bidding. □

C. *Proofs for Section III*

Proof of Lemma 5. From Lemma 1,

$$b(P|V_H) = \frac{1}{1 + \frac{1-\underline{q}(P)}{\underline{q}(P)} \times \frac{u-1}{u}} = \frac{u(P - V_L)}{(u-1)V_H - uV_L + P},$$

which is strictly decreasing in V_H since $u > 1$ and $P \geq k > V_L$. □

LEMMA A.5: *The seller's expected payoff in the due diligence subgame, $F_S(q|P, V_H)$, is continuous in q, P, and V_H. Furthermore, for $P \in (k, V_H)$, if $q < b(P)$, then $F_S(q|P, V_H)$ is increasing in V_H; and if $q \geq b(P)$ then $F_S(q|P, V_H)$ is constant in V_H.*

Proof. Continuity follows from the closed-form solutions of $F_S(z|P, V_H)$, see the proof of Lemma 1, for $z \leq \beta(P)$ and $z \geq \beta(P)$ and the value matching of these two functions at $z = \beta(P)$. For $z < \beta(P)$, the buyer performs due diligence. The seller's payoff is given by

$$F_S(z|P) = (P - k)\frac{1 + e^{\beta(P)}}{e^{u\beta(P)}} \frac{e^{uz}}{1 + e^z} < P - k.$$

By Lemma 5, increasing V_H leads to a decrease in $\beta(P)$ and, in turn, an increase in $\frac{1+e^{\beta(P)}}{e^{u\beta(P)}}$ since $u > 1$ and $\beta(P) \equiv \ln\left(\frac{b(P)}{1-b(P)}\right)$. Hence, $F_S(z|P)$ increases in V_H for $z < \beta(P)$. Finally, for $z \geq \beta(P)$, we have that $F_S(z|P) = P - k$, which is independent of V_H. $\square$

Proof of Proposition 8. If (i) holds, Lemma 5 implies (ii), and (iii) then follows by definition. To verify that there exists an equilibrium in which (i) holds, specify that the strong bidder offers $P^{st}(q_0)$, all weak bidders offer $P_S(q_0)$, the seller accepts an offer that maximizes his payoff assuming optimal play in the due diligence subgame and always breaks ties in favor of the strong bidder. Checking incentive constraints is immediate: by construction, the seller is maximizing his payoff; a weak bidder cannot alter anything by deviating; any deviation by the strong bidder leads to either her losing the asset (earning zero) or winning at a higher price (i.e., one less favorable to her).

For uniqueness, start with the case of one weak bidder. First, by Lemmas 5 and A.5 the strong bidder can guarantee herself the acquisition and, therefore, a positive payoff by offering $P_S(q_0) + \varepsilon$ for any $\varepsilon > 0$ small enough. Now fix an equilibrium candidate and let $\mathcal{P}_i$ be the support of the i-valuation bidder's strategy. Let $\mathcal{S}_i$ be F_S-image of $\mathcal{P}_i$: $\mathcal{S}_i = \{m : \exists P \in \mathcal{P}_i \text{ } with \text{ } F^i_S(P) = m\}$. Denote ℓ_i as the infimum of $\mathcal{S}_i$. If $\ell_{st} < \ell_w$, then the strong bidder's payoff would be zero, as elements of her support lose with probability one. This contradicts the established payoff bound, hence, $\ell_w \leq \ell_{st}$. Now suppose that the weak bidder wins with positive probability when playing ℓ_w. Then it must be that $\ell_w = \ell_{st}$ and each are played with positive atoms of probability. In this case, by the standard argument, the strong bidder should shift that mass of probability to a price more favorable to the seller to an arbitrarily small degree, thereby strictly increasing her probability of winning with an arbitrarily small decrease in her expected payoff conditional on winning. Hence, the weak bidder must lose for sure when playing ℓ_w, implying the weak bidder's payoff is zero in any equilibrium. It follows that $\ell_{st} \geq F_S(P_S(q_0))$, as otherwise the weak bidder could win with positive probability and earn a positive payoff by playing $P_S(q_0)$. Finally, for any offer P from the strong bidder that delivers the seller $F^{st}_S(P) > F_S(P_S(q_0))$, there exists $P' < P$ that also delivers $F^{st}_S(P') > F_S(P_S(q_0))$ and therefore also wins for sure, but at a more favorable price for the strong bidder. Hence, $\mathcal{P}_{st} = \{P^{st}(q_0)\}$, which is accepted by the seller with probability one. If there are multiple weak bidders, the argument is analogous with the single weak bidder above replaced the first-order statistic (in terms of value being offered to the seller) of the set of weak bidders. □

LEMMA A.6: *Fixing all other parameters, there exists a price $\underline{P}^{st}$ such that due diligence occurs and $P^{st}(q_0) = \underline{P}^{st}$ if and only if $q_0 < b^{st}(\underline{P}^{st})$ if either:*

i) $k \geq \bar{k}$ *for some* $\bar{k} \in (V_L, V_H)$*,*

ii) $\gamma \geq \bar{\gamma}$ *for some* $\bar{\gamma} > 0$*.*

Proof. We first derive sufficient conditions under which the equilibrium is a threshold equilibrium and then show that these conditions are satisfied when k or γ are large enough.

By Assumption 1, under the weak bidder's offer, due diligence occurs if and only if $z < \beta(P^*)$, see Proposition 1. For $z \geq \beta(P^*)$, the weak bidder's offer leads to direct execution. By Lemma 5 and Proposition 8, the strong bidder makes the same offer, and also executes immediately.

For $z < \beta(P^*)$, letting $\psi(P) \equiv (P-k)\frac{1+e^{\beta(P)}}{e^{u\beta(P)}}$ and $\psi^{st}(P) \equiv (P-k)\frac{1+e^{\beta^{st}(P)}}{e^{u\beta^{st}(P)}}$, we have

$$F_S(z|P) = \psi(P^*)\frac{e^{uz}}{1+e^z},$$

and similarly, for an offer P by the strong bidder for which $z < \beta^{st}(P)$

$$F_S^{st}(z|P) = \psi^{st}(P)\frac{e^{uz}}{1+e^z}.$$

Now assume that there is a unique $P \in (k, P^*)$ satisfying $\psi^{st}(P) = \psi(P^*)$, and denote it by $\underline{P}^{st}$. Consider $z < \beta^{st}(\underline{P}^{st})$. Because $\psi^{st}(k) = 0$ and $\psi(\underline{P}^{st}) > 0$, we have

$$\psi^{st}(P_0^{st}(z)) < \psi^{st}(\underline{P}^{st}) = \psi(P^*).$$

Therefore any direct execution offer $P \leq P_0^{st}(z)$ cannot be optimal for the seller so it must be that the strong bidder makes a due diligence offer. Consider next $z \in [\beta^{st}(\underline{P}^{st}), \beta(P^*))$.

Because $\psi^{st}(P^*) > \psi(P^*)$,

$$\psi^{st}(P_0^{st}(z)) \geq \psi^{st}(\underline{P}^{st}) = \psi(P^*).$$

Therefore the strong bidder makes a direct execution offer. This establishes the result under Assumption 1 and assuming there is a unique $\underline{P}^{st} \in (k, P^*)$ satisfying $\psi^{st}(\underline{P}^{st}) = \psi(P^*)$. It remains to show that (i) and (ii) are each sufficient to ensure these conditions. That each imply Assumption 1 is shown in Lemma 2. Hence, the remainder of the proof shows that (i) and (ii) are each sufficient for the existence of such a unique $\underline{P}^{st}$.

We begin with (i). For $P \in (k, V_H^{st})$, we have that $\frac{d}{dP}\psi^{st}(P|k)$ has the same sign as

$$Y(P,k) \equiv -P^3 - P^2(-2+u)V_H^{st} - (-1+u)(P(-1+u) - ku)\left(V_H^{st}\right)^2.$$

See that $Y(V_H, V_H) = V_H(V_H^{st} - V_H)(V_H + (u-1)V_H^{st}) > 0$. As a result, there exist a $\bar{k}$ such that $\frac{d}{dP}\psi^{st}(P|k) > 0$ for all $k \in [\bar{k}, V_H]$ and $P \in [k, V_H]$. Because ψ^{st} is continuous in P and $\psi^{st}(k) = 0 < \psi(P^*) < \psi^{st}(P^*)$, it follows that for $k \geq \bar{k}$ there exists a unique solution $\underline{P}^{st}$, establishing the claim.

For (ii), $Y(P,k) \to P^2(V_H^{st} - P) > 0$ as $\gamma \to \infty$ (i.e., as $u \to 1$). The reminder of the argument is analogous to the one given for (i) above. □

Proof of Proposition 9. In equilibrium (Proposition 8), the price is $P^{st}(q_0)$, which is the lowest price that satisfies $F_S^{st}(q_0|P^{st}(q_0), V_H^{st}) = F_S(q_0|P_S(q_0), V_H)$. Consider now an increase of V_H^{st} to $\bar{V}_H^{st}$ (but leave $P^{st}(q_0)$ unchanged). First, this increase has no effect on $F_S(q_0|P_S(q_0), V_H)$. Second, given that due diligence takes place $F_S^{st}(q_0|P)$ is strictly increasing in V_H^{st}, by Lemma A.5. Hence,

$$F_S^{st}(q_0|P^{st}(q_0), \bar{V}_H^{st}) > F_S^{st}(q_0|P^{st}(q_0), V_H^{st}) = F_S(q_0|P_S(q_0), V_H).$$

Continuity of F_S^{st} in P (Lemma A.5), the fact that $F_S^{st}(q_0|P = k) = 0$, and the definition of $P^{st}(q_0)$ implies that the lowest price $\bar{P}^{st}(q_0)$ that satisfies $F_S^{st}(q_0|\bar{P}^{st}(q_0), \bar{V}_H^{st}) = F_S(q_0|P_S(q_0), V_H)$ is strictly less than the original price, $P^{st}(q_0)$. By Proposition 8, this is the new equilibrium price. Moreover, the execution threshold is decreasing in V_H^{st} (all else equal, Lemma 5) and increasing in price (immediate from the closed-form in (2)), so falls with the increase in V_H^{st}. Finally, the payoffs to the weak bidder and the seller are unchanged by the increase in V_H^{st}, whereas the strong bidder gains from both the increase in valuation and the decrease in price. Hence, total equilibrium surplus increases with V_H^{st}. □

Proof of Proposition 10. In equilibrium (Proposition 8), the price is $P^{st}(q_0)$, which is the lowest price that satisfies $F_S^{st}(q_0|P^{st}(q_0)) = F_S(q_0|P_S(q_0), V_H)$. By hypothesis, $q_0 < b(\underline{P}^{st})$, and in turn $q_0 < b(P^*)$ by Lemma 5. Hence, $F_S(q_0|P)$ is strictly increasing in V_H, by Lemma A.5. Hence, an increase in V_H in turn increases $F_S(q_0|P_S(q_0), V_H)$. Since F^{st} is not directly affected by V_H, by definition of P^{st}, all prices at or below the equilibrium price deliver value below the now higher value of $F_S(q_0|P_S(q_0), V_H)$. Hence, the equilibrium price (Proposition 8) must increase in response to an increase in V_H. Finally, because the valuation of the acquirer is unchanged, the sole effect on the efficiency is via the time to execration. Because due diligence takes place even without the increase in V_H, and because $b^{st}(P)$ is increasing in P (immediate from the closed-form in (2)), it follows that there will be even more due diligence and lower efficiency as a result of V_H increasing. □

D. Proofs for Section IV

Proof of Proposition 11. Clearly, the candidate is an equilibrium. Since due diligence is efficient and the seller extracts all of the surplus, any other offer will either (i) be less attractive to the seller and therefore rejected, or (ii) earn negative expected payoff for the acquirer. In either case, such an offer does not constitute a profitable deviation for bidders. For uniqueness, note that with upfront unconditional payments, there is now transferable utility and thus by the standard argument for Bertrand competition, bidders must earn zero profit (otherwise they can increase U by ϵ and win with probability one). Second, for any other candidate equilibrium, there exists an offer that the seller prefers to accept and earns a strictly positive expected profit. Thus, a profitable deviation exists. □

Proof of Proposition 12. First, let $r = 0$. Given price P and break-up fee B, the acquirer solves

$$\sup_{\tau} \mathbb{E}_q \left[\max\{q_\tau V_H - P, -B\} - \tau c\right] = \sup_{\tau} \mathbb{E}_q \left[\max\left\{q_\tau V_H - (P - B), 0\right\} - \tau c\right] - B.$$

Therefore, in the flow-cost model with $r = 0$, the price/break-up fee contract $(P, B) = (P_c, B_c)$ is equivalent to the upfront unconditional transfer/price contract $(U, P) = (B_c, P_c - B_c)$. Hence, any price/break-up fee contract $(P, B) = (k + B_c, B_c)$ is socially efficient in the due diligence subgame. The seller can therefore capture all of the socially efficient total surplus (respecting the buyer's participation constraint) by setting $B_c = F^c_B(q|P = k)$.

Second, let $r > 0$. The social planner's optimal stopping problem is

$$\begin{aligned} W(q) &= \sup_{\tau} \mathbb{E}_q \left[-\int_0^\tau e^{-rt} c dt + e^{-r\tau} \max\left\{q_\tau V_H - k, 0\right\} \right] \\ &= \sup_{\tau} \mathbb{E}_q \left[e^{-r\tau} \left(\max\left\{q_\tau V_H - k, 0\right\} + \frac{c}{r} \right) \right] - \frac{c}{r}. \end{aligned}$$

Alternatively, for any price/break-up fee contract (P, B) agreed to by the seller and the acquirer, the acquirer solves the following optimal stopping problem (where the superscript c is dropped for ease of exposition):

$$\begin{aligned} F_B(q) &= \sup_{\tau} \mathbb{E}_q \left[-\int_0^\tau e^{-rt} c dt + e^{-r\tau} \max\left\{q_\tau V_H - P, -B\right\} \right] \\ &= \sup_{\tau} \mathbb{E}_q \left[-\int_0^\tau e^{-rt} \left(c - rB\right) dt + e^{-r\tau} \max\left\{q_\tau V_H - (P - B), 0\right\} - \int_0^\tau e^{-rt} rB dt - e^{-r\tau} B \right] \\ &= \sup_{\tau} \mathbb{E}_q \left[-\int_0^\tau e^{-rt} \left(c - rB\right) dt + e^{-r\tau} \max\left\{q_\tau V_H - (P - B), 0\right\} \right] - B \\ &= \sup_{\tau} \mathbb{E}_q \left[e^{-r\tau} \left(\max\left\{q_\tau V_H - (P - B), 0\right\} + \frac{c - rB}{r} \right) \right] - \frac{c - rB}{r} - B. \end{aligned}$$

Substituting in $(P, B) = (k, 0)$ yields that the two problems are identical in this case. Therefore, as in baseline model (Proposition 3), the socially optimal solution can be implemented with $(P, B) = (k, 0)$.

Observe that for the social planner's solution, the upper threshold, $\hat{b}_{SP}$, and the lower threshold, $\hat{a}_{SP} < \hat{b}_{SP}$, satisfy: $\hat{b}_{SP}V_H - k > 0$ and $\hat{a}_{SP}V_H - k < 0$. To see why, assume not. Then either $\hat{a}_{SP}V_H - k \geq 0$ or $\hat{b}_{SP}V_H - k \leq 0$. In the former case, the stopping problem has a linear reward function that is non-negative at both thresholds. Since q is a martingale, $c, r > 0$ implies it is strictly optimal to stop for all $q \in (\hat{a}_{SP}, \hat{b}_{SP})$, generating a contradiction. In the latter case, $\hat{b}_{SP}V_H - k \leq 0$, the payoff upon reaching either threshold is $W(\hat{a}_{SP}) = W(\hat{b}_{SP}) = 0$. Because $c > 0$, it is then strictly optimal to stop for all $q \in (\hat{a}_{SP}, \hat{b}_{SP})$, generating a contradiction. Hence, $W(\hat{a}_{SP}) = 0$ and $W(\hat{b}_{SP}) = \hat{b}_{SP}V_H - k$. In addition, the thresholds must satisfy the smooth-pasting conditions for optimality: $W'(\hat{b}_{SP}) = V_H$ and $W'(\hat{a}_{SP}) = 0$.

For the purpose of contradiction, suppose that some $(P, B) \neq (k, 0)$ also implemented the socially optimal thresholds. The same arguments given above imply that $\hat{b}_{SP}V_H - (P - B) > 0$ and $\hat{a}_{SP}V_H - (P - B) < 0$. Then analogous smooth-pasting conditions are necessary for acquirer optimality: $F_B'(\hat{b}_{SP}) = V_H$ and $F_B'(\hat{a}_{SP}) = 0$. Furthermore, both $F_B(q)$ and $W(q)$ satisfy the same Feynman-Kac equation for $q \in (\hat{a}_{SP}, \hat{b}_{SP})$, whose constants can be determined by the smooth-pasting conditions. Therefore, $F_B(q) = W(q)$ for $q \in [\hat{a}_{SP}, \hat{b}_{SP}]$. Immediately, then $F_B(\hat{a}_{SP}) = W(\hat{a}_{SP}) = 0$, which implies $B = 0$. Furthermore, $F_B(\hat{b}_{SP}) = W(\hat{b}_{SP}) = \hat{b}_{SP}V_H - k$, which implies $P = k$, generating a contradiction. Hence, the only (P, B) that implements the socially optimal solution is $(k, 0)$. Because the seller's payoff under $(P, B) = (k, 0)$ is zero, it is not seller optimal, completing the proof. $\square$

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Internet Appendix for "Due Diligence"

BRENDAN DALEY, THOMAS GEELEN, and BRETT GREEN*

ABSTRACT

This Internet Appendix provides information and analysis supporting the main text.

I. Formulas for Estimating the Model Extensions

In this section, we derived the formulas for probability of deal failure and expected time to completion in the two extensions of the model, which are used for estimation.

A. *Exogenous Risk of Deal Failure*

If the Poisson arrival precedes execution, then assign $\tau^* = \infty$. So, the failure probability is $1 - \mathbb{P}_0(\tau^* < \infty)$. Let $v(z) = \mathbb{P}_z(\tau^* < \infty) = \mathbb{E}_z\left[\mathbb{I}_{\{\tau^*<\infty\}}\right]$. For $z \leq \beta(P^*)$, v satisfies the following differential equation and boundary conditions

$$\begin{aligned} \lambda(0 - v(z_0)) + \frac{\phi^2}{2}\left((2p(z) - 1)v'(z) + v''(z)\right) &= 0, \\ v(\beta(P^*)) &= 1, \\ \lim_{z \to -\infty} v(z) &= 0. \end{aligned}$$

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The differential equation's solution is

$$v(z) = \frac{c_1 e^{w_1 z}}{1+e^z} + \frac{c_2 e^{w_2 z}}{1+e^z}, \quad \text{where} \;\; w_1 = \frac{1+\sqrt{1+8\frac{\lambda}{\phi^2}}}{2}, w_2 = \frac{1-\sqrt{1+8\frac{\lambda}{\phi^2}}}{2}.$$

Using the boundary conditions we get that for $z \leq \beta(P^*)$

$$v(z) = \frac{e^{w_1(z-\beta(P^*))}\left(1+e^{\beta(P^*)}\right)}{1+e^z}.$$

Calculating the expected time to completion $\mathbb{E}_z\left[\tau^*|\tau^* < \infty\right]$ is analogous to Lemma 4. Accounting for the probability of exogenous failure before date t, the probability density function becomes:

$$\begin{aligned}\mathbb{P}_z\left(\tau^* = t\right) &= p(z)\mathbb{P}_z(\tau^* = t|\theta = H) + (1-p(z))\mathbb{P}_z(\tau^* = t|\theta = L) \\ &= e^{-\lambda t}\frac{(\beta(P^*)-z)e^{-\frac{\left(t\phi^2+2(\beta(P^*)-z)\right)^2}{8t\phi^2}}}{\sqrt{2\pi}t^{3/2}\phi}\frac{1+e^{\beta(P^*)}}{1+e^z}.\end{aligned}$$

The expected time to completion is

$$\mathbb{E}_z\left[\tau^*|\tau^* < \infty\right] = \int_0^\infty t\frac{\mathbb{P}_z\left(\tau^* = t\right)}{\mathbb{P}(\tau^* < \infty)} = \frac{\max\{2(\beta(P^*)-z), 0\}}{\phi\sqrt{8\lambda+\phi^2}}.$$

B. *Flow Costs*

The failure probability is $1-\mathbb{P}_0(\tau^* < \infty)$. Let $v(z) = \mathbb{P}_z(\tau^* < \infty) = \mathbb{E}_z\left[\mathbb{I}_{\{\tau^* < \infty\}}\right]$. For $z \in [\alpha(P^*), \beta(P^*)]$, v satisfies the following differential equation and boundary conditions

$$\begin{aligned}\frac{\phi^2}{2}\left((2p(z)-1)v'(z) + v''(z)\right) &= 0, \\ v(\beta(P^*)) &= 1, \\ v(\alpha(P^*)) &= 0.\end{aligned}$$

The differential equation's solution is $v(z) = \frac{c_1}{1+e^z} + c_2$. Using the boundary conditions we get that for $z \in [\alpha(P^*), \beta(P^*)]$

$$v(z) = \frac{\left(1 + e^{\beta(P^*)}\right)\left(e^{\alpha(P^*)} - e^z\right)}{\left(e^{\alpha(P^*)} - e^{\beta(P^*)}\right)\left(1 + e^z\right)}.$$

Written in probability-space: for $q \in [a(P^*), b(P^*)]$, $v(q) = \frac{q - a(P^*)}{b(P^*) - a(P^*)}$

Next, we calculate the expected time to completion. Let $T = \min\{\tau_\beta, \tau_\alpha\}$ where $\tau_\beta = \inf\{t > 0 | Z_t \geq \beta\}$ and $\tau_\alpha = \inf\{t > 0 | Z_t \leq \alpha\}$. From Girsanov's theorem it follows that

$$\mathbb{E}_z\left[Y_T | \theta = H\right] = \mathbb{E}_z\left[e^{Z_T - z} Y_T | \theta = L\right]$$

for any $\mathcal{F}_T$ measurable Y_T. If we take $Y_T = \tau_\beta \mathbb{I}_{\{\tau_\beta = T\}}$ then we get

$$\begin{aligned}
&\mathbb{P}_z(\tau_\beta = T | \theta = H)\mathbb{E}_z\left[\tau_\beta | \theta = H, \tau_\beta = T\right] \\
=&\mathbb{E}_z\left[\tau_\beta \mathbb{I}_{\{\tau_\beta = T\}} | \theta = H\right] \\
=&\mathbb{E}_z\left[e^{Z_T - z} \tau_\beta \mathbb{I}_{\{\tau_\beta = T\}} | \theta = L\right] \\
=&\mathbb{P}_z(\tau_\beta = T | \theta = L) e^{\beta - z} \mathbb{E}_z\left[\tau_\beta | \theta = L, \tau_\beta = T\right]. \qquad \text{(IA.1)}
\end{aligned}$$

Similar to the unconditional case, we can explicitly calculate $\mathbb{P}_z(\tau_\beta = T|\theta)$. Plugging the explicit solution for $\mathbb{P}_z(\tau_\beta = T|\theta)$ into equation (IA.1) yields

$$\mathbb{E}_z\left[\tau_\beta | \theta = H, \tau_\beta = T\right] = \mathbb{E}_z\left[\tau_\beta | \theta = L, \tau_\beta = T\right],$$

which implies that $\mathbb{E}_z\left[\tau_\beta | \theta = H, \tau_\beta = T\right] = \mathbb{E}_z\left[\tau_\beta | \theta = L, \tau_\beta = T\right] = \mathbb{E}_z\left[\tau_\beta | \tau_\beta = T\right]$. Similar arguments show that $\mathbb{E}_z\left[\tau_\alpha | \theta = H, \tau_\alpha = T\right] = \mathbb{E}_z\left[\tau_\alpha | \theta = L, \tau_\alpha = T\right] = \mathbb{E}_z\left[\tau_\alpha | \tau_\alpha = T\right]$.

Using these results, we have that

$$\mathbb{E}_z\left[T | \theta = H\right] = \mathbb{P}_z(\tau_\beta = T | \theta = H)\mathbb{E}_z\left[\tau_\beta | \tau_\beta = T\right] + (1 - \mathbb{P}_z(\tau_\beta = T | \theta = H))\mathbb{E}_z\left[\tau_\alpha | \tau_\alpha = T\right],$$

$$\mathbb{E}_z\left[T | \theta = L\right] = \mathbb{P}_z(\tau_\beta = T | \theta = L)\mathbb{E}_z\left[\tau_\beta | \tau_\beta = T\right] + (1 - \mathbb{P}_z(\tau_\beta = T | \theta = L))\mathbb{E}_z\left[\tau_\alpha | \tau_\alpha = T\right].$$

Using similar arguments as for the exogenous failure model allows us to explicitly calculate $\mathbb{E}_z[T|\theta]$. This leaves us with two equations and two unknowns ($\mathbb{E}_z[\tau_\beta|\tau_\beta = T]$ and $\mathbb{E}_z[\tau_\alpha|\tau_\alpha = T]$). Only the former is used to estimate the model and is give by

$$\mathbb{E}_z[\tau_\beta|\tau_\beta = T] = \frac{2\left(\frac{2e^\alpha(\alpha-\beta)}{e^\alpha-e^\beta} + \frac{2e^\alpha(\alpha-z)}{e^z-e^\alpha} + \beta - z\right)}{\phi^2}$$

for $z \in [\alpha, \beta]$.

II. Asymmetric Information

In this section, assume that the seller knows the asset's type $\theta \in \{L, H\}$, while as before, the type is unknown to bidders. In addition, we allow the seller's reservation value to depend on the asset's type k_θ with $V_H > k_H > k_L > V_L = 0$. As in Section IV.A in the paper, we pay particular attention to contracts, $C = (U, P)$, that specify $U \geq 0$ as the unconditional upfront transfer and P as the price paid contingent on execution. Let $F_\theta(q|P) = \mathbb{E}_q^\theta\left[e^{-r\tau(P)}(P - k_\theta)\right]$ denote the value to a type-θ seller derived from the due diligence subgame with contracted price P. F_θ differs from F_S in that both the expectation about what the acquirer will learn during due diligence and the reservation value depend on θ.

A. *Constrained Efficiency*

We first revisit the problem of a social planner. As before, the first-best outcome is to execute the transaction immediately if $\theta = H$ and never if $\theta = L$. We refer to the *constrained efficient* outcome as the solution to the problem of a planner who has the same access to information as the acquirer (i.e., the planner does not know θ but can learn about it through due diligence). The planner's problem can be written as $\sup_\tau \mathbb{E}_q\left[e^{-r\tau}(V(q_\tau) - k_\theta)\right]$.

As before, it is socially optimal to delay executing the deal until beliefs reach a threshold, denoted b_{SP}, above which it is socially optimal to execute the deal. Moreover, there exists a unique contingent price $P_{SP} \in (k_L, k_H)$ under which b_{SP} is the acquirer-optimal threshold in the due diligence subgame. That is, $b(P_{SP}) = b_{SP}$.[1]

While a contingent price of P_{SP} induces constrained efficient execution, the difference in seller types creates a hurdle to the implementation of the constrained efficient outcome. Because $P_{SP} < k_H$, the high-type seller earns a loss at the time of execution. To satisfy the high-type seller's participation constraint, this loss needs to be overcome with a transfer from the acquirer. However, when the initial belief is pessimistic, even transferring the acquirer's entire expected surplus is insufficient to compensate for the high type's expected loss.

PROPOSITION IA.1 (Constrained Efficiency): *There exists a contract* (U, P) *that achieves the constrained efficient outcome and satisfies bidder and seller participation constraints if and only if*

$$q_0 \geq \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{V_H - V_L}\right\}. \tag{IA.2}$$

B. *Equilibrium*

We now analyze the game in which the seller posts a contract $C = (U, P)$. Under this formulation, the model's first stage is a signaling game due to the seller's private information.[2] We therefore modify our solution concept from subgame perfect Nash equilibrium to perfect Bayesian equilibrium (PBE).[3] In equilibrium, for any posted contract C, bidders update their belief about θ from q_0 to $\tilde{q}(C)$ that is consistent with the seller's posting strategy. If accepted, the seller's total expected payoff from $C = (U, P)$ is $U + F_\theta(\tilde{q}(C)|P)$.

As is common in signaling games, there exist many PBE, due to the flexibility afforded to off-path beliefs. We focus on the high-type optimal PBE, hereafter the *HTO equilibrium.* Below we provide sufficient conditions for the HTO equilibrium to be unique. After characterizing the HTO equilibrium, we discuss its appeal in terms of well-known equilibrium

[1]Specifically, $b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$, and $P_{SP} = \alpha V_H + (1-\alpha)V_L$, where $\alpha = \frac{k_L - V_L}{(V_H - V_L) - (k_H - k_L)}$.

[2]Our results are unchanged if instead bidders make public contract offers to the seller. While the arguments are slightly more involved in this case, the key is that the seller's choice of contract to accept still serves as a signal of his private information.

[3]See Fudenberg and Tirole (1991, pp. 331-333).

refinements.

Unlike many signaling games, the high type in our model never does best by separating.

PROPOSITION IA.2: *In any separating PBE, the transaction is never executed.*

In order to separate from the low type, the high type must post a contract so unfavorable to the bidder that she never executes *even though* the acquirer is sure the seller has a high type asset (otherwise the low type would prefer to imitate the high type). Of course, in a separating equilibrium, the low type does not transact either. Henceforth, we refer to such equilibria, in which the transaction is never executed, as *trivial*.

Proposition IA.2 implies that a HTO equilibrium must involve at least some pooling. In fact, it involves full pooling.

LEMMA IA.1: *Any HTO equilibrium involves full pooling on a single contract.*

Because a HTO equilibrium is full pooling, the bidders' belief after the offer is posted remains their prior, q_0, and the contract chosen solves

$$\max_{U,P} \; U + F_H(q_0|P)$$
$$s.t. \quad F_B(q_0|P) - U \geq 0.$$

The constraint ensures that bidders are willing to accept the offer. Clearly, the constraint binds at any solution. In addition, if immediate execution is optimal, then there is no distinction between the contingent and upfront transfers (only $U + P$ matters). Hence it is without loss to set $P = P_0(q_0)$ in this case, reducing the problem to

$$\max_{P \geq P_0(q_0)} F_H(q_0|P) + F_B(q_0|P). \tag{IA.3}$$

The following assumption guarantees that the solution to (IA.3) is unique.

ASSUMPTION IA.1: $\frac{k_H}{V_H} \geq \left(1 - \frac{u}{2}\right)$.

We generalize our results when Assumption IA.1 does not hold in Proposition IA.4.

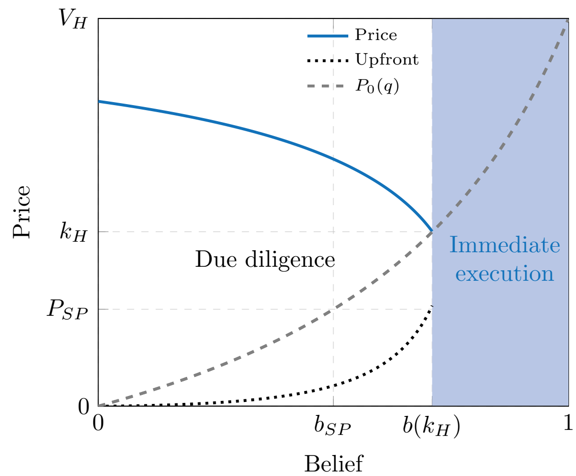

Figure IA.1. Equilibrium contract with asymmetric information. This figure is consistent with Assumption IA.1.

PROPOSITION IA.3: *Under Assumption IA.1, the HTO equilibrium is unique. Moreover,*

(i) if $q_0 < b(k_H)$, the price is strictly above k_H and the acquirer conducts due diligence.

(ii) if $q_0 \geq b(k_H)$, there is immediate execution.

Compared to the constrained efficient outcome, the HTO equilibrium features a higher price and too much due diligence. The HTO equilibrium also requires a higher initial belief in order to forgo due diligence and immediately execute the deal. Intuitively, the greater price benefits the high type, who expects good news and to complete the deal more quickly, at the expense of the low type. Unlike in the baseline model, this inefficiency persists even with the unconditional upfront transfer.

Also in contrast to the baseline model, the high-type optimal price depends on the prior belief as illustrated in Figure IA.1. Perhaps surprisingly, the high-type optimal price is decreasing in the belief. The intuition is that the high type faces a tradeoff between generating total surplus (which is maximized at $P = P_{SP}$), and extracting surplus from the low type with a higher price. When bidders believe the low type is more likely, extraction becomes relatively more important.

Assumption IA.1 fails if and only if $k_H < V_H/2$ and γ is sufficiently large, in which case we have the following characterization.

PROPOSITION IA.4: *If Assumption IA.1 fails, there exists a pair* $(\overline{q}_H, \underline{q}_H)$, $\overline{q}_H \geq \underline{q}_H > b(k_H) > b_{SP}$, *such that the HTO equilibrium is unique for all* $q_0 \notin (\underline{q}_H, \overline{q}_H)$. *Moreover,*

(i) if $q_0 < \underline{q}_H$, *the price is above* k_H *and the acquirer conducts due diligence.*

(ii) if $q_0 > \overline{q}_H$, *there is immediate execution.*

Under Assumption IA.1, $\overline{q}_H = \underline{q}_H = b(k_H)$. So, there are two differences without Assumption IA.1. First, we only have proven that $\overline{q}_H \geq \underline{q}_H$, though in all of the numerical examples we have analyzed, $\overline{q}_H = \underline{q}_H$. Second, and of more economic interest, with a higher quality information process, the high type induces due diligence for a strictly larger set of priors.

We conclude with the connection between refinements and HTO equilibria.

PROPOSITION IA.5: *A nontrivial PBE is a HTO equilibrium if and only if it satisfies both Divinity (Banks and Sobel, 1987) and the Undefeated Criterion (Mailath et al., 1993).*[4]

C. Proofs for Section II

Proof of Proposition IA.1. Recall that in the due diligence subgame the constrained efficient outcome is achieved if and only if $P = P_{SP}$. The payoffs to the acquirer and the seller under a contract (U, P_{SP}) are $F_B(q_0|P_{SP}) - U$ and $F_\theta(q_0|P_{SP}) + U$, respectively. Notice that $F_B(q_0|P_{SP}) \geq 0$ because the acquirer has the right not to execute, and $F_L(q_0|P_{SP}) \geq 0$ because $P_{SP} > k_L$. Hence, there exits a $U \geq 0$ such that (U, P_{SP}) achieves the constrained efficient outcome and satisfies bidder and (both) seller participation constraints if and only if $F_H(q_0|P_{SP}) + F_B(q_0|P_{SP}) \geq 0$. The remainder of the proof establishes that this inequality is equivalent to (IA.2).

First, we need to show that there exists a $\hat{q}$ such that $F(q|P_{SP}) \equiv F_H(q|P_{SP}) + F_B(q|P_{SP}) > 0$ if and only if $q > \hat{q}$. The if part is immediate: since $b_{SP} < 1$, for q close enough to 1, the

[4]The *intuitive criterion* (Cho and Kreps, 1987)—the canonical refinement used to generate separation in signaling games—has no refining power in our game because the signal in our model (i.e., posting a contract) has no direct cost. For example, no matter how large a price the seller demands, the worst that can happen is that the contract is rejected, resulting in a payoff of zero.

transaction is executed immediately and $F(q|P_{SP}) = V(q) - k_H \approx V_H - k_H > 0$.

To see that F is negative for low q, assume that $q < b_{SP}$, then

$$F(q|P_{SP}) =$$

$$(q-1)\left(\frac{1}{1-q}\right)^u q^{u-1} \underbrace{\frac{(k_H - V_H)\left(\frac{u(k_H V_L - k_L V_H)}{(u-1)(k_H - V_H)(V_H - V_L)}\right)^{1-u}(-k_H u + k_L u + q(V_H - V_L))}{u(-k_H + k_L + V_H - V_L)}}_{\equiv m(q)}$$

with $\lim_{q\to 0} m(q) > 0$. Therefore, there exists a $\tilde{q} > 0$ such that for $q \in (0, \tilde{q})$ $F(q|P_{SP})$ is negative.

For $q < b_{SP}$, $F(q|P_{SP})$ can cross zero only once, namely when $m(q) = 0$ which has a unique solution. For $q \geq b_{SP}$, $F(q|P_{SP}) = V(q) - k_H$ can also only cross zero once. Furthermore, $F(q|P_{SP})$ is continuous for all q. Hence, $F(q|P_{SP})$ can cross zero at most twice.

Continuity, in combination with the fact that F is negative for sufficiently low q and positive for sufficiently high q, implies that $F(q|P_{SP})$ must cross zero an odd number of times. Thus, $F(q|P_{SP})$ crosses zero exactly once and from below: there exists a $\hat{q} \in (0,1)$ such that $F(q|P_{SP}) > 0$ if and only if $q > \hat{q}$.

There are two possible cases for $\hat{q}$ now:

1. If $\hat{q} \geq b_{SP}$, then $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$ and

$$\frac{k_H - V_L}{V_H - V_L} \geq b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}$$

   which is the case if and only if

$$u \geq \frac{k_H - V_L}{k_H - k_L}$$

   since b_{SP} is decreasing in u (less informative due diligence means sooner execution).

2. If $\hat{q} < b_{SP}$, then $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$ and

$$\frac{u(k_H - k_L)}{V_H - V_L} < b_{SP} = \frac{u(V_L - k_L)}{k_H(u-1) - u(k_L + V_H - V_L) + V_H}. \tag{IA.4}$$

Observe that

$$k_H(u-1) - u(k_L + V_H - V_L) + V_H = (k_H - V_H)(u-1) - u(k_L - V_L) < 0.$$

Therefore, (IA.4) holds if and only if

$$\frac{u(k_H - k_L)}{V_H - V_L}(k_H(-1+u) + V_H - u(k_L + V_H - V_L)) - u(V_L - k_L) > 0.$$

which is a quadratic equation in u with as solution $u = 0$ and $u = \frac{k_H - V_L}{k_H - k_L}$. Furthermore, the terms in front of u^2 are negative which implies that for $u \in \left(0, \frac{k_H - V_L}{k_H - k_L}\right)$, we have $\frac{u(k_H - k_L)}{V_H - V_L} < b_{SP}$. As a result for $u < \frac{k_H - V_L}{k_H - k_L}$ we must have that $\hat{q} < b_{SP}$.

To conclude, we know that if $u < \frac{k_H - V_L}{k_H - k_L}$ holds then $\hat{q} < b_{SP}$ and $\hat{q} = \frac{u(k_H - k_L)}{V_H - V_L}$ while if it fails then $\hat{q} = \frac{k_H - V_L}{V_H - V_L}$. This is equivalent to saying $\hat{q} = \min\left\{\frac{u(k_H - k_L)}{V_H - V_L}, \frac{k_H - V_L}{k_H - k_L}\right\}$. $\square$

Taking as given the acquirer's solution to the due diligence subgame, and resulting value functions F_B, F_H, F_L, we consider the first-stage signaling game. Let the seller's utility from posting a contract $C = (U, P)$, which results in a belief $\tilde{q}(C)$ be $u_\theta(C, \tilde{q}(C))$. If the contract is accepted, $u_\theta(C, \tilde{q}(C)) = U + F_\theta(\tilde{q}(C)|P)$; if it is rejected $u_\theta(C, \tilde{q}(C)) = 0$. A posted contract will be accepted if and only if $U \leq F_B(\tilde{q}(C)|P)$. Let $\mathbb{S}_\theta$ be the support of the type-θ seller's strategy.

We use σ to denote an arbitrary strategy profile accompanied by the on-path beliefs uniquely determined via Bayes rule, and in a slight abuse of notation, $u_\theta(\sigma)$ to be the payoff of the type-θ seller in σ. We say that σ is an equilibrium if there exist off-path beliefs that support σ by (i) creating no incentives for any player to deviate and (ii) satisfying the requirements for PBE (Fudenberg and Tirole, 1991, pp. 331-333).

LEMMA IA.2: *In any PBE σ: (i) if $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, then σ is trivial; and (ii) $u_L(\sigma) = 0$ if and only if σ is trivial.*

Proof. For (i), fix a PBE σ such that there exists $C \in \mathbb{S}_L$, but $C \notin \mathbb{S}_H$. Then $\tilde{q}(C) = 0$. Hence, on the path, if C is posted, the deal will never be executed. This is because there are negative gains from trade when $\theta = L$: if there were positive probability of trade, then $F_L(0|P) + F_B(0|P) < 0$, and (since U is just a transfer) at least one player earns a negative payoff and would do better to deviate. So, $u_L(\sigma) = 0$, and σ is trivial by (ii).

For (ii), it is immediate that if σ is trivial, then $u_L(\sigma) = 0$. Now suppose that σ is nontrivial. Then there is positive probability of deal execution when $\theta = H$. So there is some $C' = (U', P') \in \mathbb{S}_H$ that is accepted. Hence,

$$u_H(\sigma) = u_H(C', \tilde{q}(C')) = U' + \mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_H) \geq 0,$$

with $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$. Moreover, $\mathbb{E}^H_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] \geq \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right] > 0$, where the first inequality follows from $\tau(P')$ being a threshold policy and the second inequality from $\mathcal{Q}^H$ and $\mathcal{Q}^L$ being equivalent measures. Therefore, with $U' \geq 0$ and $k_L < k_H$, we have

$$u_L(\sigma) \geq u_L(C', \tilde{q}(C')) = U' + \mathbb{E}^L_{\tilde{q}(C')}\left[e^{-r\tau(P')}\right](P' - k_L) > 0.$$

□

Proof of Proposition IA.2. By definition, in any separating PBE, $\mathbb{S}_L \cap \mathbb{S}_H = \emptyset$. Hence, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the result follows from Lemma IA.2. □

The High-type Optimal (HTO) Contract To begin, we characterize the high-type optimal equilibrium restricting to pure-strategy, full pooling PBE. We then prove Lemma IA.1 using a series of auxillary lemmas. Propositions IA.3-IA.5 follow.

In any pure-strategy, full pooling PBE: $\mathbb{S}_L = \mathbb{S}_H = \{C\}$ for some contract $C = (U, P)$, and $\tilde{q}(C) = q_0$. As discussed in the text, high-type optimality requires that in the contract C

we have $U = F_B(q_0|P)$ and P solves (IA.3). We refer to such solutions as *high-type optimal (pooling) contracts*, and use P_H to denote the price component of the solution. As with the proofs for the baseline model, analysis is aided by working with the log-likelihood of belief.

First, we show that if $z < \beta(k_H)$ then $P_H(z) > k_H$ and there is inefficient delay in equilibrium (Lemma IA.3), as in the baseline model . This result directly implies that there exists a $\underline{z}_H^* \geq \beta(k_H)$ such that for $z < \underline{z}_H^*$ (too much) delay takes place in equilibrium. Second, we show that there exists a $\bar{z}_H^*$ such that for $z \geq \bar{z}_H^*$ direct completion of the deal is optimal (Lemma IA.4). Finally, we prove existence of a lower bound on the high-type seller's reservation value k_H, which if satisfied implies $\bar{z}_H^* = \underline{z}_H^* = \beta(k_H)$ (Lemma IA.5).

LEMMA IA.3: *In a high-type optimal contract, if $q_0 < b(k_H)$ then $P_H(q_0) > k_H$, and hence, execution is delayed. That is, $b(P_H(q_0)) > q_0$.*

Proof. First, define the high-type's payoff for $z < \beta(P)$ as

$$\begin{aligned} F(z|P) &\equiv F_H(z|P) + F_B(z|P) \\ &= (P - k_H)e^{(u-1)z}\left(\frac{Pu}{(u-1)(V_H - P)}\right)^{1-u} + \frac{Pe^{uz}\left(\frac{Pu}{(u-1)(V_H-P)}\right)^{-u}}{(u-1)\left(e^z+1\right)} \end{aligned}$$

with $u > 1$ and $f(z|P) \equiv \frac{\partial}{\partial P}F(z|P)$. The proof is handled in three steps corresponding to three different regions of the parameter space. Figure IA.2 illustrates the region of the parameter space for which each different step proves that $f(z|P) > 0$.

STEP 1: *For $z < \beta(k_H)$, we have that $f(z|P_0(z)) > 0$.*

Ovserve that F is constant in P for all $P < P_0(z)$. Hence, we can verify the claim by evaluating f at $z = \beta(P)$. Because $u > 1$, direct calculation yields

$$f(\beta(P)|P) = \frac{(u-1)V_H(P - k_H)}{P(P - V_H)} > 0.$$

STEP 2: *For $z < \beta(k_H)$, we have that $f(z|k_H) > 0$.*

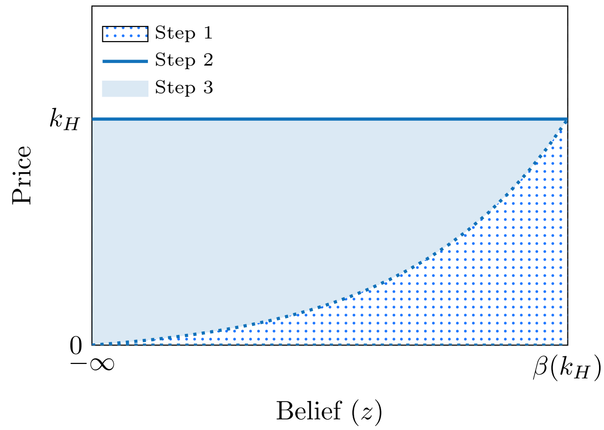

Figure IA.2. Illustration of where in the state space the different steps prove that $f(z|P) > 0$.

The proof of the previous step also implies that $f(\beta(k_H)|k_H) = 0$. For $z < \beta(k_H)$, we have that

$$f(z|k_H) = \frac{e^{(u-1)z}\left(\frac{k_H u}{(u-1)(V_H-k_H)}\right)^{-u}((u-1)e^z(k_H - V_H) + k_H u)}{(u-1)\left(e^z+1\right)\left(V_H - k_H\right)},$$

Notice f has the same sign as $(u-1)e^z(k_H - V_H) + k_H u$, which is decreasing in z. Therefore, $f(z|k_H)$ has exactly one root and is positive for $z < \beta(k_H)$.

STEP 3: *For any $z < \beta(k_H)$, $f(z|P)$ has at most one root in P on $P \in (P_0(z), k_H)$.*

Proving this step has several sub-steps:

i) For $z < \beta(P)$ and $P < V_H$, $f(z|P)$ has the same sign as

$$M(z,P) = -\Big((u-1)e^z\left(V_H(V_H - k_H u) + P^2 + P(u-2)V_H\right) + u\left(V_H(k_H(-u) + k_H + P(u-2)) + P^2\right)\Big). \quad \text{(IA.5)}$$

M is quadratic in P, and therefore has at most two roots in P. Furthermore, M is negative as as $P \to \pm\infty$ since the coefficient on P^2 is negative.

ii) Evaluating M at $P = k_H$, we get

$$M(z,k_H) = (k_H - V_H)\left(-\left((u-1)e^z(k_H - V_H) + k_H u\right)\right),$$

which is decreasing in z. Furthermore, $M(\beta(k_H), k_H) = 0$. Hence $M(z, k_H)$ is positive for $z < \beta(k_H)$. From (i), $M(z, P)$ is negative for P sufficiently large or small. It follows that $M(z, P)$ has exactly one root for $P < k_H$ and therefore at most one root for $P \in (0, k_H)$.

From Step 1 and Step 2, we know $f(z|P)$ is positive at $P = P_0(z)$ and at $P = k_H$. Step 3 tells us $f(z|P)$ can change signs at most once on the interval $P \in (P_0(z), k_H)$. Combined, we conclude that $f(z|P)$ does not change signs on the interval and therefore is strictly positive for all $P \in (P_0(z), k_H)$, which proves Lemma IA.3.

Remark: For $z < b(k_H)$ and $P \in (P_0(z), k_H)$ $f(z|P) = 0$ cannot occur because this would imply $M(z, P)$ is tangent at zero and does not cross it, which is impossible since it is a quadratic equation with negative limits as $P \to \pm\infty$ and $M(z, k_H) > 0$. □

LEMMA IA.4: *There exists a $\bar{z}_H^*$ such that for $z \geq \bar{z}_H^*$ direct completion is the high-type optimal (pooling) contract.*

Proof. The proof has three steps. First, $F(z|P) \leq \sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)]$.

This follows from

$$\begin{aligned} F(z|P) &= \mathbb{E}_z^H[e^{-r\tau(P)}(P - k_H)] + \mathbb{E}_z[e^{-r\tau(P)}(V(q_{\tau(P)}) - P)] \\ &\leq \mathbb{E}_z^H[e^{-r\tau(P)}(P - k_H)] + \mathbb{E}_z^H[e^{-r\tau(P)}(V(q_{\tau(P)}) - P)] \\ &= \mathbb{E}_z^H[e^{-r\tau(P)}(V(q_\tau) - k_H)] \\ &\leq \sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)], \end{aligned}$$

where the first inequality comes from the fact that the drift of Z is higher under the high-type's probability measure than the acquirer's.

Second, the solution to $\sup_\tau \mathbb{E}_z^H[e^{-r\tau}(V(q_\tau) - k_H)]$ is a threshold policy: stop if and only if $z > z_H^*$. The proof of this result is standard and therefore omitted (see e.g., Dixit et al., 1994, Ch. 5). Third, for $z \geq z_H^*$, the high type optimal contract can achieve the upper bound from the first step with $P = P_0(z)$. □

LEMMA IA.5: *Assume* $\frac{k_H}{V_H} \geq \left(1 - \frac{u}{2}\right)$. *Then* $\bar{z}_H^* = \underline{z}_H^* = b(k_H)$.

Proof. We know that for $z < \beta(P)$ and $P < V_H$, $f(z|P)$ has the same sign as $M(z, P)$, which is quadratic in P (see (IA.5)). Recall from the Proof of Lemma IA.3 that $M(z, k_H)$ is decreasing in z and $M(\beta(k_H), k_H) = 0$. Therefore, for $z > \beta(k_H)$, $M(z, k_H)$ is negative.

Observe that

$$\frac{\partial M(z,P)}{\partial P} = -\left((u-1)e^z + u\right)(2P + (u-2)V_H),$$

which is negative for $P > k_H$ because $2P + (u-2)V_H > 2k_H + (u-2)V_H > 0$. Therefore, for $z > \beta(k_H)$ and $P > P_0(z) > k_H$ we have that $M(z, P) < 0$ and therefore that $f(z|P) < 0$. This result implies that $P > P_0(z)$ is never optimal for $z > \beta(k_H)$.

Lemma IA.3 in combination with the previous step completes the proof. □

The High-type Optimal (HTO) Equilibrium

Denote the set of high-type optimal (pooling) contracts as $\mathbb{C}_H(q_0)$ and the high type's PBE payoff from pooling on a solution as $u_H^*(q_0)$. If a contract will result in immediate execution, then the distinction between U and P is irrelevant as only $U + P$ matters. In this case, we simplify exposition by setting $P = P_0(q_0)$. If there exist contracts that induce immediate execution given q_0, define $C_i(q_0) \equiv (P_0(q_0), \mathbb{E}_{q_0}[V_\theta] - P_0(q_0))$ as the high-type optimal contract that results in immediate execution. If it exists, define $C_d(q_0) \equiv \arg\max_{\{C:P>P_0(q_0)\}} U + F_H(q_0|P)$, which is the high-type optimal contract that results in delay given q_0.

LEMMA IA.6: *The set of high-type optimal contracts has the following properties:*

1. *If* $C_d(q_0)$ *exists, then it is unique.*

2. $\mathbb{C}_H(q_0)$ *is one of the following:*

   (a) $\mathbb{C}_H(q_0) = \{C_i(q_0)\}$, *i.e., direct execution is uniquely optimal.*

   (b) $\mathbb{C}_H(q_0) = \{C_d(q_0)\}$, *i.e, delay is uniquely optimal*

   (c) $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$.

3. *If* $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$, *then* $u_L(C_i(q_0), q_0)) > u_L(C_d(q_0), q_0))$.

4. For any $C \in \mathbb{C}_H(q_0)$, $u_\theta(C, q) > 0$ is nondecreasing in q.

5. The problem's value $u^*_H(q_0)$ is increasing in q_0.

Proof. Taking each in turn:

1. From the proof of Lemma IA.3 it follows that the first-order derivative of the seller's payoff (assuming there is delay $q < b(P)$) has the same sign as a function that is a second-order polynomial in P. This implies that for $P > P_0(q_0)$ there can be at most one local maximum for $P \in (P_0(q_0), V_H)$. The fact that there is one local maximum for $P > P_0(q_0)$ proves the result

2. This result follows directly from the previous result.

3. We know that

$$\begin{aligned} u_L(C_i(q_0), q_0) + (k_L - k_H) &= u_H(C_i(q_0), q_0) \\ &= u_H(C_d(q_0), q_0) \\ &> u_L(C_d(q_0), q_0) + \mathbb{E}^L_{q_0}\left[e^{-r\tau(P)}(k_L - k_H)\right]. \end{aligned}$$

The first equality follows from the fact that payoffs only differ because of the different reservation values. The second equality follows from the fact that the high-type seller is indifferent, and the third equality follows from the fact that $P \geq k_H > k_L$ and therefore worse dynamics of beliefs make the seller worse off (the upfront is fixed). This result implies that

$$u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0) + \mathbb{E}^L_{q_0}\left[e^{-r\tau(P)}(k_L - k_H)\right] - (k_L - k_H) > u_L(C_d(q_0), q_0).$$

The last inequality follows from the fact that $\tau(P) > 0$ and $k_H > k_L$ and therefore

$$\mathbb{E}^L_{q_0}\left[e^{-r\tau(P)}(k_L - k_H)\right] > (k_L - k_H).$$

4. Because $C \in \mathbb{C}_H(q_0)$, we know that $P \geq k_H > k_L$ and therefore the seller's payoff upon completion is non-negative. The upfront is fixed. Therefore, higher beliefs q weakly improve $u_\theta(C, q)$.

   The fact that $P \geq k_H > k_L$ implies that the seller's payoff (without the upfront) is non-negative. Furthermore, the upfront is always positive since $P < V_H$ and therefore $u_\theta(C, q) > 0$.

5. Take any $C \in \mathbb{C}_H(q_0)$. Fixing the price $P < V_H$, the upfront is increasing in beliefs. Furthermore, the seller's expected payoff upon execution is non-decreasing in beliefs since $P \geq k_H > k_L$ therefore the seller must be better of with higher beliefs. □

DEFINITION IA.1: *Let* $\mathbb{H}(q_0)$ *be the set of pure-strategy, full pooling PBE in which both types pool on some contract* $C \in \mathbb{C}_H(q_0)$.

LEMMA IA.7: $\mathbb{H}(q_0)$ *is the set of HTO equilibria.*

Proof. It is straightforward to verify that any $\sigma^* \in \mathbb{H}(q_0)$ is a PBE when paired with off-path beliefs $\tilde{q}(C') = 0$ for all deviations C'. Turning to the payoffs, by definition, $u_H(\sigma^*) = u_H^*(q_0) > 0$ for any $\sigma^* \in \mathbb{H}(q_0)$. Consider now PBE $\sigma \notin \mathbb{H}(q_0)$. Let $\underline{q}_\theta = \min_{\mathbb{S}_\theta} \tilde{q}(C)$. If $\underline{q}_H < q_0$, then there exists $C \in \mathbb{S}_H$ such that,

$$u_H(\sigma) = u_H(C, \underline{q}_H) \leq u_H^*(\underline{q}_H) < u_H^*(q_0),$$

where the last inequality is from Lemma IA.6(5). If $\underline{q}_H > q_0$, then Bayesian consistency requires $\underline{q}_L = 0$. Therefore, $\mathbb{S}_L \not\subseteq \mathbb{S}_H$, and the equilibrium is trivial by Lemma IA.2, so $u_H(\sigma) = 0 < u_H^*(q_0)$. This establishes that $u_H(\sigma) < u_H^*(q_0)$ for any PBE σ in which $\underline{q}_H \neq q_0$.

So, if $u_H(\sigma) \geq u_H^*(q_0)$, then $\underline{q}_H = q_0$. Bayesian consistency then requires $\tilde{q}(C) = q_0$ for all $C \in \mathbb{S}_H \cup \mathbb{S}_L$, meaning both types play the same strategy. By definition, to achieve $u_H^*(q_0)$ their common support must be a subset of $\mathbb{C}_H(q_0)$. If $\mathbb{C}_H(q_0)$ is a singleton, this concludes the proof. Otherwise, by Lemma IA.6(2), $\mathbb{C}_H(q_0) = \{C_i(q_0), C_d(q_0)\}$. In this case, the seller cannot mix between the two solutions, because $u_L(C_i(q_0), q_0) > u_L(C_d(q_0), q_0)$ by Lemma IA.6(3). □

Proof of Lemma IA.1. Follows from Lemma IA.7. □

Proof of Proposition IA.3. Follows from Lemmas IA.3-IA.5 and IA.7. □

Proof of Proposition IA.4. Follows from Lemmas IA.3, IA.4, and IA.7. □

Proof of Proportion IA.5. Lemma IA.7 establishes that $\mathbb{H}(q_0)$ is the set of HTO equilibria. To establish Proportion IA.5, we use the following specification of off-path beliefs. Define $B_\theta(C, u) \equiv \{q : u_\theta(C, q) \geq u\}$. For any $\sigma \in \mathbb{H}(q_0)$ with $C^* \in \mathbb{C}_H(q_0)$ as the pooling contract, specify the off-path beliefs as follows. For any $C' \neq C^*$: $\tilde{q}(C') = q_0$ if $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, and $\tilde{q}(C') = 0$ otherwise, where $\subset$ denotes strict inclusion. To verify σ remains a PBE, note that neither type wishes to deviate to C' with $\tilde{q}(C') = 0$. In addition, the high type has no incentive to deviate to C' with $\tilde{q}(C') = q_0$ as $u_H(C', \tilde{q}(C')) = u_H(C', q_0) \leq u_H^*(q_0) = u_H(\sigma)$. By construction, at such C', $B_L(C', u_L(\sigma)) \subset B_H(C', u_H(\sigma))$, so the low type has no incentive to deviate either. Moreover, since $u_H(\sigma) = u_H^*(q_0) > 0$ (by Lemma IA.6(4)) the equilibrium is nontrivial, and the beliefs are constructed to satisfy divinity.

The remainder of the proof is handled by Lemmas IA.8-IA.10 below. □

LEMMA IA.8: *If $\sigma \in \mathbb{H}(q_0)$, then σ is undefeated.*

Proof. Let $\sigma \in \mathbb{H}(q_0)$ with off-path beliefs as specified at the start of the proof of the proposition. For the purpose of contradiction, suppose there exists PBE σ' that defeats σ. By Lemma IA.7, $u_H(\sigma) \geq u_H(\sigma')$. The following are then required for σ' to defeat σ: there exist $C' \in \mathbb{S}'_L$ where $C' \neq C^*$, and $u_L(\sigma') > u_L(\sigma) \geq 0$. Hence, by Lemma IA.2, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C' \in \mathbb{S}'_H$ as well. For σ' to defeat σ then further requires that $u_H(\sigma') \geq u_H(\sigma)$. So, we have $u_H(\sigma') = u_H(\sigma)$. In this case, σ' defeating σ requires that $\tilde{q}(C') \notin [0, q_0]$. However, as specified above, $\tilde{q}(C) \in [0, q_0]$ for all C, which is a contradiction. □

LEMMA IA.9: *If there exist PBEs $\sigma \in \mathbb{H}(q_0)$ and $\sigma' \notin \mathbb{H}(q_0)$, such that $u_L(\sigma') > u_L(\sigma)$, then σ' does not satisfy Divinity.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* \in \mathbb{C}_H(q_0)$. From Lemma IA.6(4), $u_\theta(C^*, q)$ is nondecreasing in q for both θ. Hence, for both types, $B_\theta(C^*, u_\theta(\sigma'))$ is an interval $[\underline{b}_\theta, 1]$, where $\underline{b}_\theta$ denotes the lowest q-value such that $u_H(C^*, \underline{b}_\theta) \geq u_\theta(\sigma')$. In addition, $\underline{b}_H \leq q_0 < \underline{b}_L$, where the first inequality is from the hypothesis that $\sigma' \notin \mathbb{H}(q_0)$ and the second inequality is from the hypothesis that $u_L(\sigma') > u_L(\sigma)$. If C^* is off-path under σ', Divinity then requires that $\tilde{q}(C^*) \geq q_0$ and the high type would profit by deviating to C^*, breaking the PBE. Hence, it is sufficient to establish that C^* is off-path under σ'.

Suppose C^* is on-path in σ'. By $u_L(\sigma') > u_L(\sigma) > 0$ and Lemma IA.2, σ' is nontrivial, so $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and $u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$. From Lemma IA.6(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma IA.2), which is a contradiction. $\square$

LEMMA IA.10: *If there exist nontrivial PBEs $\sigma \in \mathbb{H}(q_0)$ and $\sigma' \notin \mathbb{H}(q_0)$, such that $u_L(\sigma') \leq u_L(\sigma)$, then σ that defeats σ'.*

Proof. By definition, $\mathbb{S}_H = \mathbb{S}_L = \{C^*\}$ for some $C^* = (U^*, P^*) \in \mathbb{C}_H(q_0)$. By definition, σ defeats σ' if: (i) C^* is off-path in σ'; (ii) $u_\theta(\sigma) \geq u_\theta(\sigma')$ for both θ, and holding strictly for at least one θ; and (iii) $\tilde{q}'(C^*) \notin [q_0, 1]$. Requirement (ii) holds for $\theta = H$ by Lemma IA.7 and for $\theta = L$ by hypothesis. Moreover, if requirement (iii) fails, then because $u_H(C^*, q)$ is nondecreasing in q (Lemma IA.6(4)),

$$u_H(\sigma') \geq u_H(C^*, \tilde{q}'(C^*)) \geq u_H(C^*, q_0) = u_H(\sigma),$$

which contradicts Lemma IA.7. Hence, it is sufficient to show that (i): C^* is off-path in σ'.

Suppose C^* is on-path in σ'. Since σ' is nontrivial, $\mathbb{S}'_L \subseteq \mathbb{S}'_H$. Hence, $C^* \in \mathbb{S}'_H$ and

$u_H(\sigma') = u_H(C^*, \tilde{q}'(C^*)) < u_H(\sigma) = u_H(C^*, q_0)$, where the inequality is from Lemma IA.7. From Lemma IA.6(4), $u_H(C^*, q)$ is nondecreasing in q, implying $\tilde{q}'(C^*) < q_0$. Bayesian consistency then implies that $C^* \in \mathbb{S}'_L$ and $u_L(\sigma') = u_L(C^*, \tilde{q}'(C^*))$. However, $\tilde{q}'(C^*) < q_0$ also implies that $F_B(\tilde{q}'(C^*)|P^*) < F_B(q_0|P^*) = U^*$, and C^* is rejected in σ'. It follows that $u_L(\sigma') = u_H(\sigma') = 0$, which implies that σ' is trivial (by Lemma IA.2), which is a contradiction. □

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